# 14 | FLUID MECHANICS

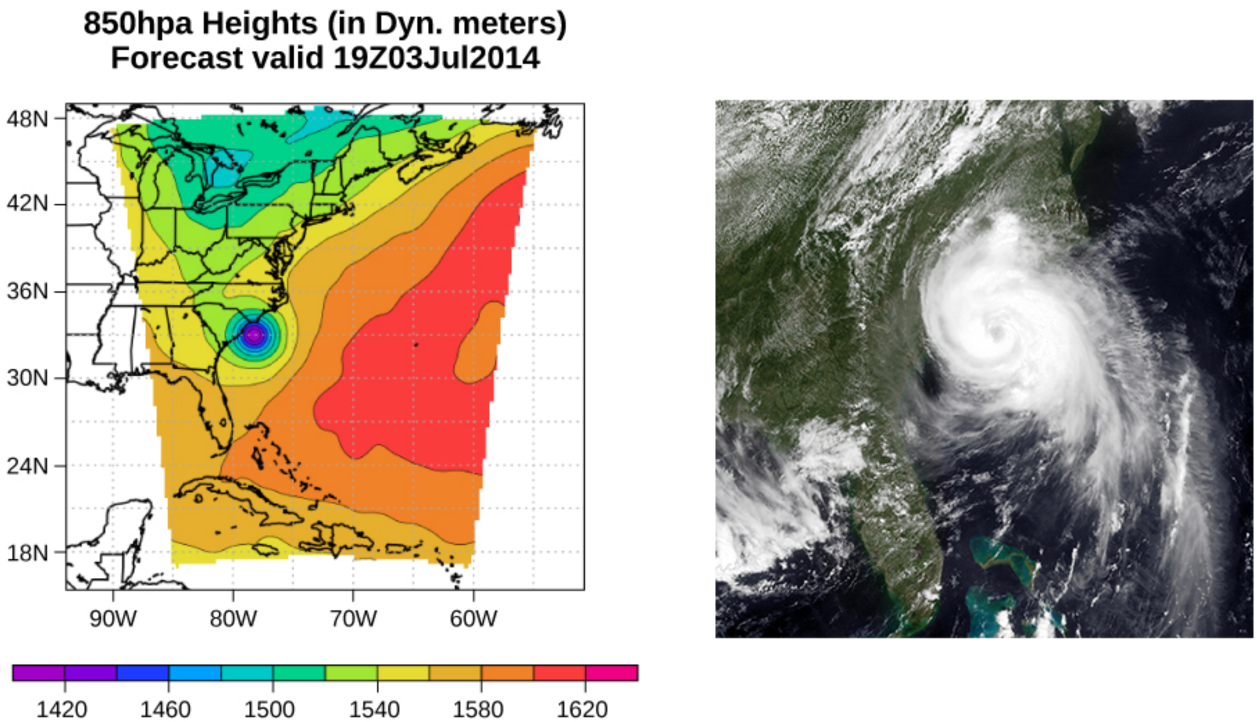



**Figure 14.1** This pressure map (left) and satellite photo (right) were used to model the path and impact of Hurricane Arthur as it traveled up the East Coast of the United States in July 2014. Computer models use force and energy equations to predict developing weather patterns. Scientists numerically integrate these time-dependent equations, along with the energy budgets of long- and short-wave solar energy, to model changes in the atmosphere. The pressure map on the left was created using the Weather Research and Forecasting Model designed at the National Center for Atmospheric Research. The colors represent the height of the 850-mbar pressure surface. (credit left: modification of work by The National Center for Atmospheric Research; credit right: modification of work by NRL Monterey Marine Meteorology Division, The National Oceanic and Atmospheric Administration)

## Chapter Outline

14.1 Fluids, Density, and Pressure
14.2 Measuring Pressure
14.3 Pascal's Principle and Hydraulics
14.4 Archimedes' Principle and Buoyancy
14.5 Fluid Dynamics
14.6 Bernoulli's Equation
14.7 Viscosity and Turbulence

## Introduction

Picture yourself walking along a beach on the eastern shore of the United States. The air smells of sea salt and the sun warms your body. Suddenly, an alert appears on your cell phone. A tropical depression has formed into a hurricane. Atmospheric pressure has fallen to nearly 15% below average. As a result, forecasters expect torrential rainfall, winds in excess of 100 mph, and millions of dollars in damage. As you prepare to evacuate, you wonder: How can such a small drop in pressure lead to such a severe change in the weather?

Pressure is a physical phenomenon that is responsible for much more than just the weather. Changes in pressure cause ears to “pop” during takeoff in an airplane. Changes in pressure can also cause scuba divers to suffer a sometimes fatal disorder known as the “bends,” which occurs when nitrogen dissolved in the water of the body at extreme depths returns to a gaseous state in the body as the diver surfaces. Pressure lies at the heart of the phenomena called buoyancy, which causes hot air balloons to rise and ships to float. Before we can fully understand the role that pressure plays in these phenomena, we need to discuss the states of matter and the concept of density.

# 14.1 | Fluids, Density, and Pressure

## Learning Objectives

By the end of this section, you will be able to:

- State the different phases of matter
- Describe the characteristics of the phases of matter at the molecular or atomic level
- Distinguish between compressible and incompressible materials
- Define density and its related SI units
- Compare and contrast the densities of various substances
- Define pressure and its related SI units
- Explain the relationship between pressure and force
- Calculate force given pressure and area

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common phases of matter. We will look at each of these phases in detail in this section.

## Characteristics of Solids

Solids are rigid and have specific shapes and definite volumes. The atoms or molecules in a solid are in close proximity to each other, and there is a significant force between these molecules. Solids will take a form determined by the nature of these forces between the molecules. Although true solids are not incompressible, it nevertheless requires a large force to change the shape of a solid. In some cases, the force between molecules can cause the molecules to organize into a lattice as shown in **Figure 14.2**. The structure of this three-dimensional lattice is represented as molecules connected by rigid bonds (modeled as stiff springs), which allow limited freedom for movement. Even a large force produces only small displacements in the atoms or molecules of the lattice, and the solid maintains its shape. Solids also resist shearing forces. (Shearing forces are forces applied tangentially to a surface, as described in **Static Equilibrium and Elasticity**.)

## Characteristics of Fluids

Liquids and gases are considered to be **fluids** because they yield to shearing forces, whereas solids resist them. Like solids, the molecules in a liquid are bonded to neighboring molecules, but possess many fewer of these bonds. The molecules in a liquid are not locked in place and can move with respect to each other. The distance between molecules is similar to the distances in a solid, and so liquids have definite volumes, but the shape of a liquid changes, depending on the shape of its container. Gases are not bonded to neighboring atoms and can have large separations between molecules. Gases have neither specific shapes nor definite volumes, since their molecules move to fill the container in which they are held (**Figure 14.2**).

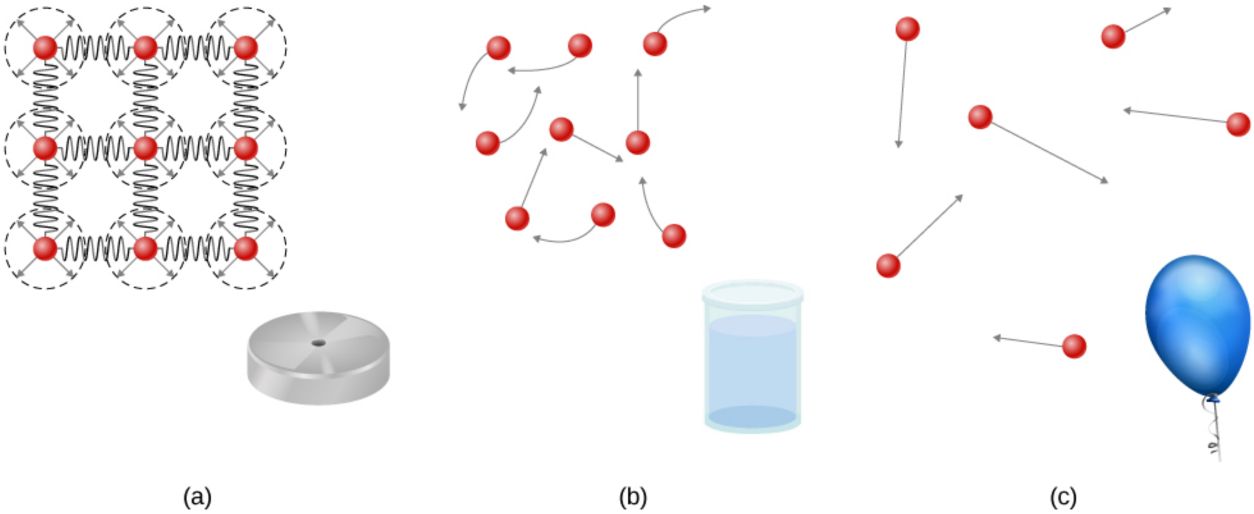



**Figure 14.2** (a) Atoms in a solid are always in close contact with neighboring atoms, held in place by forces represented here by springs. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between the atoms strongly resist attempts to compress the atoms. (c) Atoms in a gas move about freely and are separated by large distances. A gas must be held in a closed container to prevent it from expanding freely and escaping.

Liquids deform easily when stressed and do not spring back to their original shape once a force is removed. This occurs because the atoms or molecules in a liquid are free to slide about and change neighbors. That is, liquids flow (so they are a type of fluid), with the molecules held together by mutual attraction. When a liquid is placed in a container with no lid, it remains in the container. Because the atoms are closely packed, liquids, like solids, resist compression; an extremely large force is necessary to change the volume of a liquid.

In contrast, atoms in gases are separated by large distances, and the forces between atoms in a gas are therefore very weak, except when the atoms collide with one another. This makes gases relatively easy to compress and allows them to flow (which makes them fluids). When placed in an open container, gases, unlike liquids, will escape.

In this chapter, we generally refer to both gases and liquids simply as fluids, making a distinction between them only when they behave differently. There exists one other phase of matter, plasma, which exists at very high temperatures. At high temperatures, molecules may disassociate into atoms, and atoms disassociate into electrons (with negative charges) and protons (with positive charges), forming a plasma. Plasma will not be discussed in depth in this chapter because plasma has very different properties from the three other common phases of matter, discussed in this chapter, due to the strong electrical forces between the charges.

## Density

Suppose a block of brass and a block of wood have exactly the same mass. If both blocks are dropped in a tank of water, why does the wood float and the brass sink (**Figure 14.3**)? This occurs because the brass has a greater density than water, whereas the wood has a lower density than water.

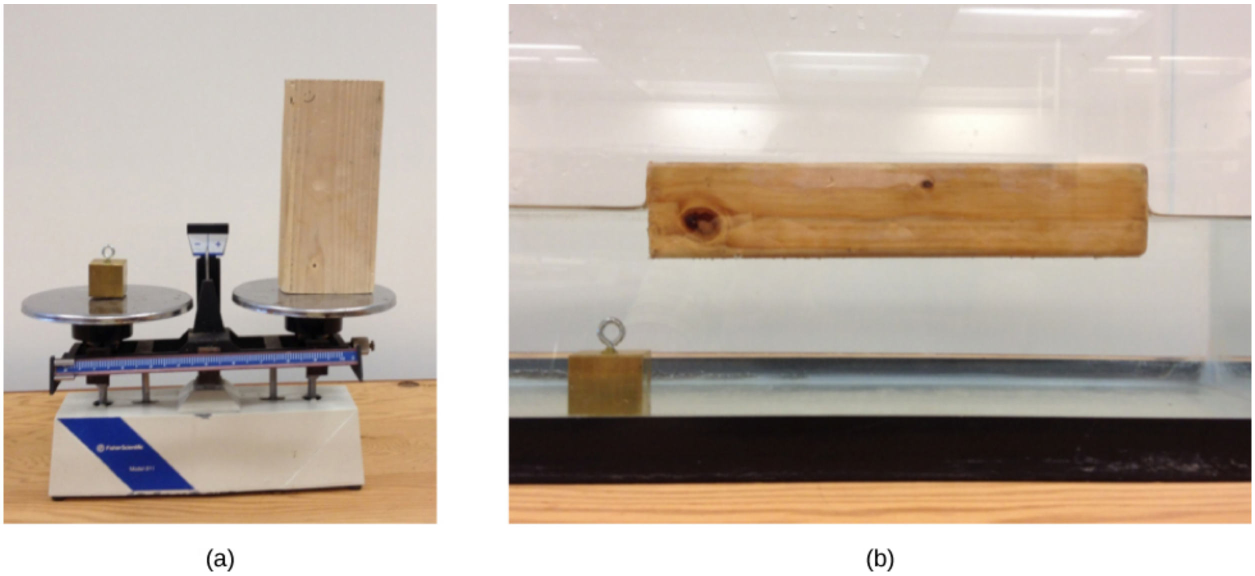


Figure 14.3 (a) A block of brass and a block of wood both have the same weight and mass, but the block of wood has a much greater volume. (b) When placed in a fish tank filled with water, the cube of brass sinks and the block of wood floats. (The block of wood is the same in both pictures; it was turned on its side to fit on the scale.)

**Density** is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid.

### Density

The average density of a substance or object is defined as its mass per unit volume,

$$\rho = \frac{m}{V} \tag{14.1}$$

where the Greek letter $\rho$ (rho) is the symbol for density, $m$ is the mass, and $V$ is the volume.

The SI unit of density is $\text{kg/m}^3$. Table 14.1 lists some representative values. The cgs unit of density is the gram per cubic centimeter, $\text{g/cm}^3$, where

$$1\ \text{g/cm}^3 = 1000\ \text{kg/m}^3.$$

The metric system was originally devised so that water would have a density of $1\ \text{g/cm}^3$, equivalent to $10^3\ \text{kg/m}^3$. Thus, the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of $1000\ \text{cm}^3$.

| Solids ($0.0°\text{C}$) | | Liquids ($0.0°\text{C}$) | | Gases ($0.0°\text{C}$, 101.3 kPa) | |
|---|---|---|---|---|---|
| Substance | $\rho(\text{kg/m}^3)$ | Substance | $\rho(\text{kg/m}^3)$ | Substance | $\rho(\text{kg/m}^3)$ |
| Aluminum | $2.70 \times 10^3$ | Benzene | $8.79 \times 10^2$ | Air | $1.29 \times 10^0$ |
| Bone | $1.90 \times 10^3$ | Blood | $1.05 \times 10^3$ | Carbon dioxide | $1.98 \times 10^0$ |
| Brass | $8.44 \times 10^3$ | Ethyl alcohol | $8.06 \times 10^2$ | Carbon monoxide | $1.25 \times 10^0$ |

Table 14.1 Densities of Some Common Substances

| Solids ($0.0°C$) | | Liquids ($0.0°C$) | | Gases ($0.0°C$, 101.3 kPa) | |
|---|---|---|---|---|---|
| Concrete | $2.40 \times 10^3$ | Gasoline | $6.80 \times 10^2$ | Helium | $1.80 \times 10^{-1}$ |
| Copper | $8.92 \times 10^3$ | Glycerin | $1.26 \times 10^3$ | Hydrogen | $9.00 \times 10^{-2}$ |
| Cork | $2.40 \times 10^2$ | Mercury | $1.36 \times 10^4$ | Methane | $7.20 \times 10^{-2}$ |
| Earth's crust | $3.30 \times 10^3$ | Olive oil | $9.20 \times 10^2$ | Nitrogen | $1.25 \times 10^0$ |
| Glass | $2.60 \times 10^3$ | | | Nitrous oxide | $1.98 \times 10^0$ |
| Gold | $1.93 \times 10^4$ | | | Oxygen | $1.43 \times 10^0$ |
| Granite | $2.70 \times 10^3$ | | | | |
| Iron | $7.86 \times 10^3$ | | | | |
| Lead | $1.13 \times 10^4$ | | | | |
| Oak | $7.10 \times 10^2$ | | | | |
| Pine | $3.73 \times 10^2$ | | | | |
| Platinum | $2.14 \times 10^4$ | | | | |
| Polystyrene | $1.00 \times 10^2$ | | | | |
| Tungsten | $1.93 \times 10^4$ | | | | |
| Uranium | $1.87 \times 10^3$ | | | | |

**Table 14.1 Densities of Some Common Substances**

As you can see by examining **Table 14.1**, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space. The gases are displayed for a standard temperature of $0.0°C$ and a standard pressure of 101.3 kPa, and there is a strong dependence of the densities on temperature and pressure. The densities of the solids and liquids displayed are given for the standard temperature of $0.0°C$ and the densities of solids and liquids depend on the temperature. The density of solids and liquids normally increase with decreasing temperature.

**Table 14.2** shows the density of water in various phases and temperature. The density of water increases with decreasing temperature, reaching a maximum at $4.0°C$, and then decreases as the temperature falls below $4.0°C$. This behavior of the density of water explains why ice forms at the top of a body of water.

| Substance | $\rho (kg/m^3)$ |
|---|---|
| Ice $(0°C)$ | $9.17 \times 10^2$ |
| Water $(0°C)$ | $9.998 \times 10^2$ |
| Water $(4°C)$ | $1.000 \times 10^3$ |

**Table 14.2 Densities of Water**

| Substance | $\rho(\mathbf{kg/m^3})$ |
|---|---|
| Water $(20°C)$ | $9.982 \times 10^2$ |
| Water $(100°C)$ | $9.584 \times 10^2$ |
| Steam $(100°C, 101.3\,kPa)$ | $1.670 \times 10^2$ |
| Sea water $(0°C)$ | $1.030 \times 10^3$ |

**Table 14.2 Densities of Water**

The density of a substance is not necessarily constant throughout the volume of a substance. If the density is constant throughout a substance, the substance is said to be a homogeneous substance. A solid iron bar is an example of a homogeneous substance. The density is constant throughout, and the density of any sample of the substance is the same as its average density. If the density of a substance were not constant, the substance is said to be a heterogeneous substance. A chunk of Swiss cheese is an example of a heterogeneous material containing both the solid cheese and gas-filled voids. The density at a specific location within a heterogeneous material is called *local density*, and is given as a function of location, $\rho = \rho(x, y, z)$ (**Figure 14.4**).

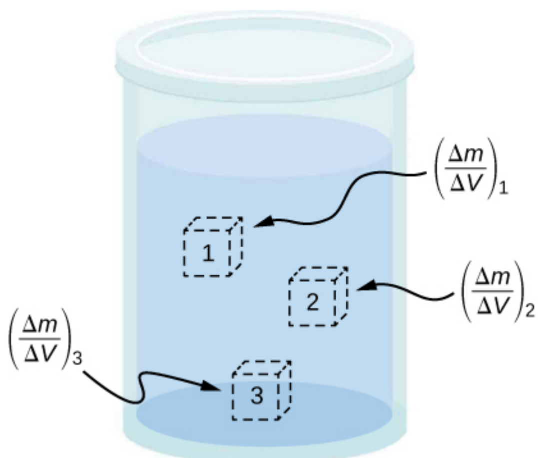



**Figure 14.4** Density may vary throughout a heterogeneous mixture. Local density at a point is obtained from dividing mass by volume in a small volume around a given point.

Local density can be obtained by a limiting process, based on the average density in a small volume around the point in question, taking the limit where the size of the volume approaches zero,

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \tag{14.2}$$

where $\rho$ is the density, $m$ is the mass, and $V$ is the volume.

Since gases are free to expand and contract, the densities of the gases vary considerably with temperature, whereas the densities of liquids vary little with temperature. Therefore, the densities of liquids are often treated as constant, with the density equal to the average density.

Density is a dimensional property; therefore, when comparing the densities of two substances, the units must be taken into consideration. For this reason, a more convenient, dimensionless quantity called the **specific gravity** is often used to compare densities. Specific gravity is defined as the ratio of the density of the material to the density of water at $4.0\,°C$ and one atmosphere of pressure, which is $1000\,kg/m^3$:

$$\text{Specific} \quad \text{ravity} = \frac{\text{Density of material}}{\text{Density of water}}.$$

The comparison uses water because the density of water is $1 \text{ g/cm}^3$, which was originally used to define the kilogram. Specific gravity, being dimensionless, provides a ready comparison among materials without having to worry about the unit of density. For instance, the density of aluminum is 2.7 in $\text{g/cm}^3$ (2700 in $\text{kg/m}^3$), but its specific gravity is 2.7, regardless of the unit of density. Specific gravity is a particularly useful quantity with regard to buoyancy, which we will discuss later in this chapter.

## Pressure

You have no doubt heard the word ‘pressure’ used in relation to blood (high or low blood pressure) and in relation to weather (high- and low-pressure weather systems). These are only two of many examples of pressure in fluids. (Recall that we introduced the idea of pressure in **Static Equilibrium and Elasticity**, in the context of bulk stress and strain.)

### Pressure

**Pressure** ($p$) is defined as the normal force $F$ per unit area $A$ over which the force is applied, or

$$p = \frac{F}{A}. \tag{14.3}$$

To define the pressure at a specific point, the pressure is defined as the force $dF$ exerted by a fluid over an infinitesimal element of area $dA$ containing the point, resulting in $p = \frac{dF}{dA}$.

A given force can have a significantly different effect, depending on the area over which the force is exerted. For instance, a force applied to an area of $1 \text{ mm}^2$ has a pressure that is 100 times as great as the same force applied to an area of $1 \text{ cm}^2$. That is why a sharp needle is able to poke through skin when a small force is exerted, but applying the same force with a finger does not puncture the skin (**Figure 14.5**).

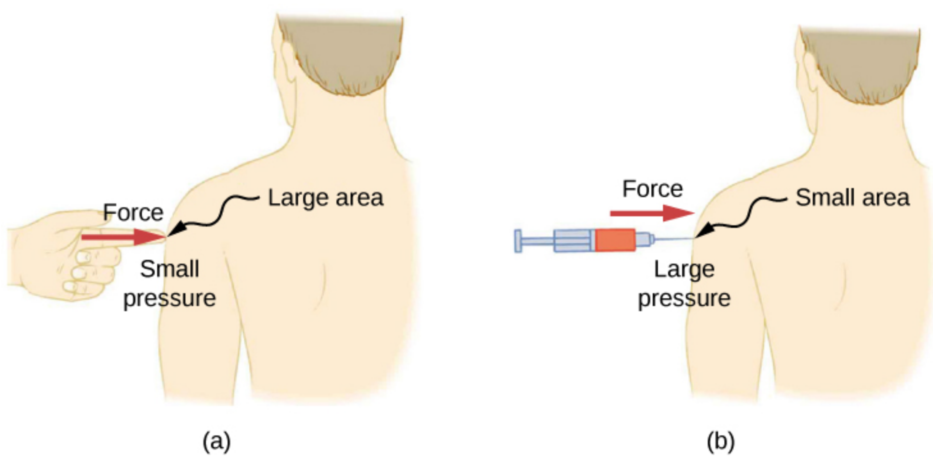



**Figure 14.5** (a) A person being poked with a finger might be irritated, but the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is enough to break the skin.

Note that although force is a vector, pressure is a scalar. Pressure is a scalar quantity because it is defined to be proportional to the magnitude of the force acting perpendicular to the surface area. The SI unit for pressure is the *pascal* (Pa), named after the French mathematician and physicist Blaise Pascal (1623–1662), where

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

Several other units are used for pressure, which we discuss later in the chapter.

### Variation of pressure with depth in a fluid of constant density

Pressure is defined for all states of matter, but it is particularly important when discussing fluids. An important characteristic of fluids is that there is no significant resistance to the component of a force applied parallel to the surface of a fluid. The molecules of the fluid simply flow to accommodate the horizontal force. A force applied perpendicular to the surface compresses or expands the fluid. If you try to compress a fluid, you find that a reaction force develops at each point inside the fluid in the outward direction, balancing the force applied on the molecules at the boundary.

Consider a fluid of constant density as shown in **Figure 14.6**. The pressure at the bottom of the container is due to the pressure of the atmosphere $(p_0)$ plus the pressure due to the weight of the fluid. The pressure due to the fluid is equal to the weight of the fluid divided by the area. The weight of the fluid is equal to its mass times the acceleration due to gravity.

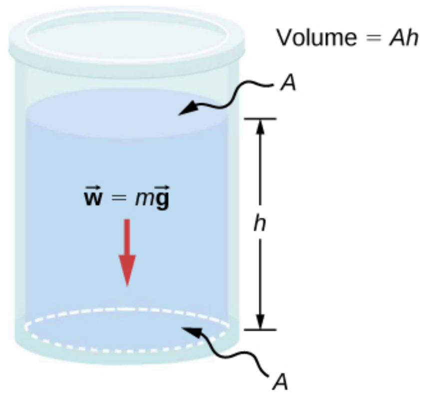



**Figure 14.6** The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), so the bottom must support it all.

Since the density is constant, the weight can be calculated using the density:

$$w = mg = \rho V g = \rho A h g.$$

The pressure at the bottom of the container is therefore equal to atmospheric pressure added to the weight of the fluid divided by the area:

$$p = p_0 + \frac{\rho A h g}{A} = p_0 + \rho h g.$$

This equation is only good for pressure at a depth for a fluid of constant density.

#### Pressure at a Depth for a Fluid of Constant Density

The pressure at a depth in a fluid of constant density is equal to the pressure of the atmosphere plus the pressure due to the weight of the fluid, or

$$p = p_0 + \rho h g, \tag{14.4}$$

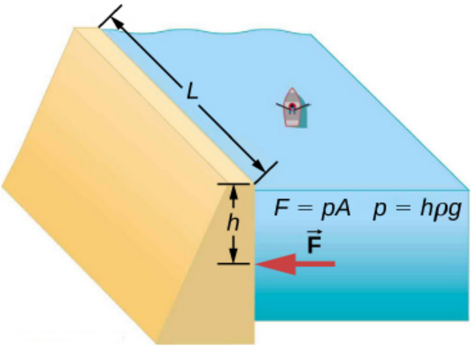
Where $p$ is the pressure at a particular depth, $p_0$ is the pressure of the atmosphere, $\rho$ is the density of the fluid, $g$ is the acceleration due to gravity, and $h$ is the depth.

**Figure 14.7** The Three Gorges Dam, erected on the Yangtze River in central China in 2008, created a massive reservoir that displaced more than one million people. (credit: "Le Grand Portage"/Flickr)

## Example 14.1

### What Force Must a Dam Withstand?

Consider the pressure and force acting on the dam retaining a reservoir of water (**Figure 14.7**). Suppose the dam is 500-m wide and the water is 80.0-m deep at the dam, as illustrated below. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam.




The average pressure $p$ due to the weight of the water is the pressure at the average depth $h$ of 40.0 m, since pressure increases linearly with depth. The force exerted on the dam by the water is the average pressure times the area of contact, $F = pA$.

### solution

a. The average pressure due to the weight of a fluid is

$$p = h\rho g. \tag{14.5}$$

Entering the density of water from **Table 14.1** and taking $h$ to be the average depth of 40.0 m, we obtain

$$\begin{aligned} p &= (40.0\,\text{m})\left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.80\frac{\text{m}}{\text{s}^2}\right) \\ &= 3.92 \times 10^5\,\frac{\text{N}}{\text{m}^2} = 392\,\text{kPa}. \end{aligned}$$

b. We have already found the value for $p$. The area of the dam is

$$A = 80.0\,\text{m} \times 500\,\text{m} = 4.00 \times 10^4\,\text{m}^2,$$

so that

$$\begin{aligned} F &= (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2) \\ &= 1.57 \times 10^{10} \text{ N}. \end{aligned}$$

### Significance

Although this force seems large, it is small compared with the $1.96 \times 10^{13}$ N weight of the water in the reservoir. In fact, it is only 0.0800% of the weight.

**14.1 Check Your Understanding** If the reservoir in Example 14.1 covered twice the area, but was kept to the same depth, would the dam need to be redesigned?

## Pressure in a static fluid in a uniform gravitational field

A *static fluid* is a fluid that is not in motion. At any point within a static fluid, the pressure on all sides must be equal—otherwise, the fluid at that point would react to a net force and accelerate.

The pressure at any point in a static fluid depends only on the depth at that point. As discussed, pressure in a fluid near Earth varies with depth due to the weight of fluid above a particular level. In the above examples, we assumed density to be constant and the average density of the fluid to be a good representation of the density. This is a reasonable approximation for liquids like water, where large forces are required to compress the liquid or change the volume. In a swimming pool, for example, the density is approximately constant, and the water at the bottom is compressed very little by the weight of the water on top. Traveling up in the atmosphere is quite a different situation, however. The density of the air begins to change significantly just a short distance above Earth's surface.

To derive a formula for the variation of pressure with depth in a tank containing a fluid of density $\rho$ on the surface of Earth, we must start with the assumption that the density of the fluid is not constant. Fluid located at deeper levels is subjected to more force than fluid nearer to the surface due to the weight of the fluid above it. Therefore, the pressure calculated at a given depth is different than the pressure calculated using a constant density.

Imagine a thin element of fluid at a depth $h$, as shown in Figure 14.8. Let the element have a cross-sectional area $A$ and height $\Delta y$. The forces acting upon the element are due to the pressures $p(y)$ above and $p(y + \Delta y)$ below it. The weight of the element itself is also shown in the free-body diagram.

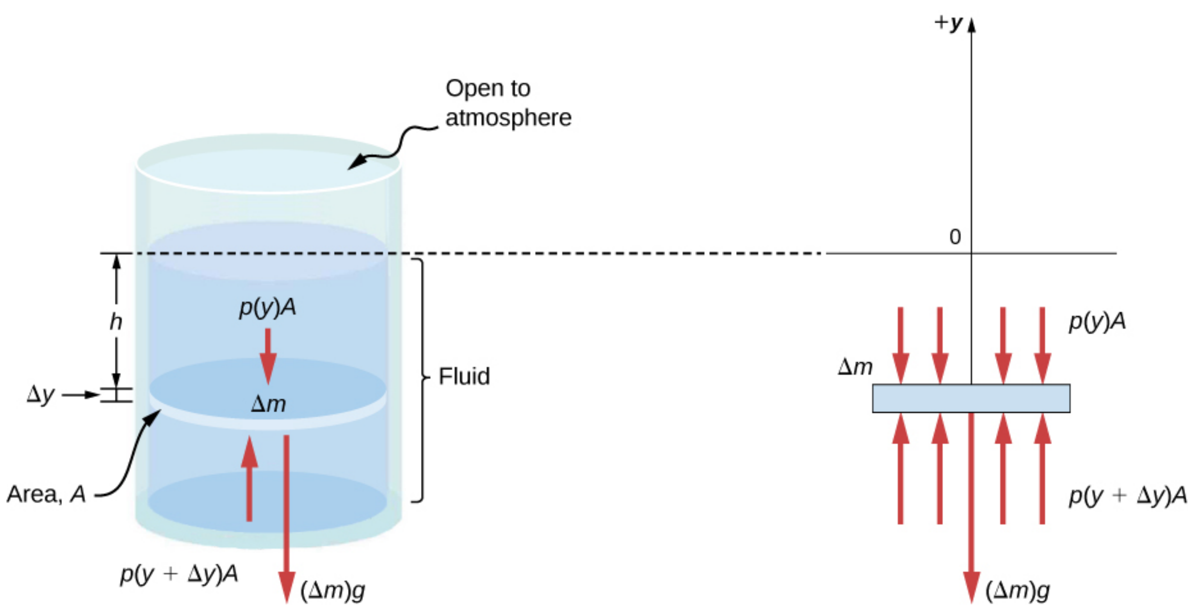



Figure 14.8 Forces on a mass element inside a fluid. The weight of the element itself is shown in the free-body diagram.

Since the element of fluid between $y$ and $y + \Delta y$ is not accelerating, the forces are balanced. Using a Cartesian $y$-axis oriented up, we find the following equation for the $y$-component:

$$p(y + \Delta y)A - p(y)A - g\Delta m = 0(\Delta y > 0). \quad (14.6)$$

Note that if the element had a non-zero $y$-component of acceleration, the right-hand side would not be zero but would instead be the mass times the $y$-acceleration. The mass of the element can be written in terms of the density of the fluid and the volume of the elements:

$$\Delta m = |\rho A \Delta y| = -\rho A \Delta y \quad (\Delta y > 0).$$

Putting this expression for $\Delta m$ into **Equation 14.6** and then dividing both sides by $A\Delta y$, we find

$$\frac{p(y + \Delta y) - p(y)}{\Delta y} = -\rho g. \quad (14.7)$$

Taking the limit of the infinitesimally thin element $\Delta y \to 0$, we obtain the following differential equation, which gives the variation of pressure in a fluid:

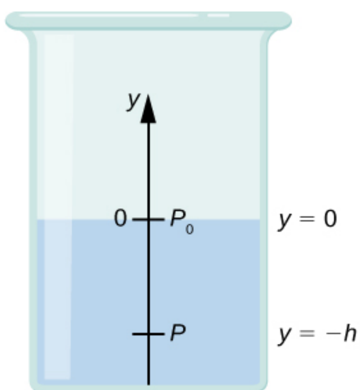
$$\frac{dp}{dy} = -\rho g. \quad (14.8)$$

This equation tells us that the rate of change of pressure in a fluid is proportional to the density of the fluid. The solution of this equation depends upon whether the density $\rho$ is constant or changes with depth; that is, the function $\rho(y)$.

If the range of the depth being analyzed is not too great, we can assume the density to be constant. But if the range of depth is large enough for the density to vary appreciably, such as in the case of the atmosphere, there is significant change in density with depth. In that case, we cannot use the approximation of a constant density.

### Pressure in a fluid with a constant density

Let's use **Equation 14.9** to work out a formula for the pressure at a depth $h$ from the surface in a tank of a liquid such as water, where the density of the liquid can be taken to be constant.




We need to integrate **Equation 14.9** from $y = 0$, where the pressure is atmospheric pressure $(p_0)$, to $y = -h$, the $y$-coordinate of the depth:

$$\begin{aligned} \int_{p_0}^{p} dp &= -\int_{0}^{-h} \rho g dy \\ p - p_0 &= \rho g h \\ p &= p_0 + \rho g h. \end{aligned} \quad (14.9)$$

Hence, pressure at a depth of fluid on the surface of Earth is equal to the atmospheric pressure plus $\rho gh$ if the density of the fluid is constant over the height, as we found previously.

Note that the pressure in a fluid depends only on the depth from the surface and not on the shape of the container. Thus, in a container where a fluid can freely move in various parts, the liquid stays at the same level in every part, regardless of the shape, as shown in **Figure 14.9**.

**Figure 14.9** If a fluid can flow freely between parts of a container, it rises to the same height in each part. In the container pictured, the pressure at the bottom of each column is the same; if it were not the same, the fluid would flow until the pressures became equal.

### Variation of atmospheric pressure with height

The change in atmospheric pressure with height is of particular interest. Assuming the temperature of air to be constant, and that the ideal gas law of thermodynamics describes the atmosphere to a good approximation, we can find the variation of atmospheric pressure with height, when the temperature is constant. (We discuss the ideal gas law in a later chapter, but we assume you have some familiarity with it from high school and chemistry.) Let $p(y)$ be the atmospheric pressure at height $y$. The density $\rho$ at $y$, the temperature $T$ in the Kelvin scale (K), and the mass $m$ of a molecule of air are related to the absolute pressure by the ideal gas law, in the form

$$p = \rho\frac{k_B T}{m} \text{(atmosphere)}, \tag{14.10}$$

where $k_B$ is Boltzmann's constant, which has a value of $1.38 \times 10^{-23}$ J/K.

You may have encountered the ideal gas law in the form $pV = nRT$, where $n$ is the number of moles and $R$ is the gas constant. Here, the same law has been written in a different form, using the density $\rho$ instead of volume $V$. Therefore, if pressure $p$ changes with height, so does the density $\rho$. Using density from the ideal gas law, the rate of variation of pressure with height is given as

$$\frac{dp}{dy} = -p\left(\frac{mg}{k_B T}\right),$$

where constant quantities have been collected inside the parentheses. Replacing these constants with a single symbol $\alpha$, the equation looks much simpler:

$$\begin{aligned}
\frac{dp}{dy} &= -\alpha p \\
\frac{dp}{p} &= -\alpha dy \\
\int_{p_0}^{p(y)} \frac{dp}{p} &= \int_0^y -\alpha dy \\
[\ln(p)]_{p_0}^{p(y)} &= [-\alpha y]_0^y \\
\ln(p) - \ln(p_0) &= -\alpha y \\
\ln\left(\frac{p}{p_0}\right) &= -\alpha y
\end{aligned}$$

This gives the solution

$$p(y) = p_0 \exp(-\alpha y).$$

Thus, atmospheric pressure drops exponentially with height, since the $y$-axis is pointed up from the ground and $y$ has positive values in the atmosphere above sea level. The pressure drops by a factor of $\frac{1}{e}$ when the height is $\frac{1}{\alpha}$, which gives us a physical interpretation for $\alpha$: The constant $\frac{1}{\alpha}$ is a length scale that characterizes how pressure varies with height and is often referred to as the pressure scale height.

We can obtain an approximate value of $\alpha$ by using the mass of a nitrogen molecule as a proxy for an air molecule. At temperature $27\ °\text{C}$, or 300 K, we find

$$\alpha = -\frac{mg}{k_B T} = \frac{4.8 \times 10^{-26}\ \text{kg} \times 9.81\ \text{m/s}^2}{1.38 \times 10^{-23}\ \text{J/K} \times 300\ \text{K}} = \frac{1}{8800\ \text{m}}.$$

Therefore, for every 8800 meters, the air pressure drops by a factor 1/*e*, or approximately one-third of its value. This gives us only a rough estimate of the actual situation, since we have assumed both a constant temperature and a constant *g* over such great distances from Earth, neither of which is correct in reality.

### Direction of pressure in a fluid

Fluid pressure has no direction, being a scalar quantity, whereas the forces due to pressure have well-defined directions: They are always exerted perpendicular to any surface. The reason is that fluids cannot withstand or exert shearing forces. Thus, in a static fluid enclosed in a tank, the force exerted on the walls of the tank is exerted perpendicular to the inside surface. Likewise, pressure is exerted perpendicular to the surfaces of any object within the fluid. **Figure 14.10** illustrates the pressure exerted by air on the walls of a tire and by water on the body of a swimmer.

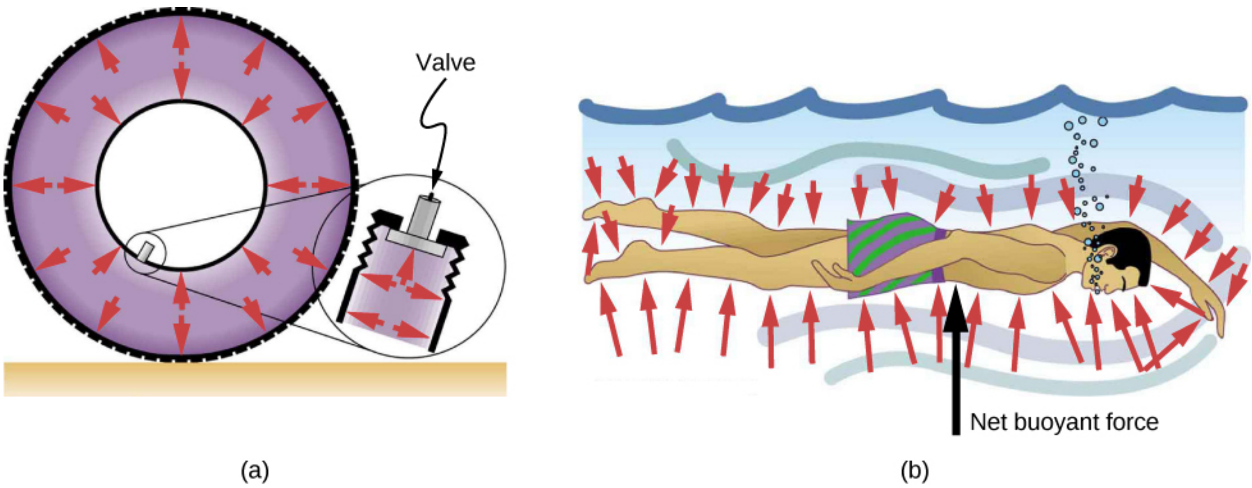



**Figure 14.10** (a) Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows represent directions and magnitudes of the forces exerted at various points. (b) Pressure is exerted perpendicular to all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force. The net vertical force on the swimmer is equal to the sum of the buoyant force and the weight of the swimmer.

# 14.2 | Measuring Pressure

## Learning Objectives

By the end of this section, you will be able to:

- Define gauge pressure and absolute pressure
- Explain various methods for measuring pressure
- Understand the working of open-tube barometers
- Describe in detail how manometers and barometers operate

In the preceding section, we derived a formula for calculating the variation in pressure for a fluid in hydrostatic equilibrium. As it turns out, this is a very useful calculation. Measurements of pressure are important in daily life as well as in science and engineering applications. In this section, we discuss different ways that pressure can be reported and measured.

## Gauge Pressure vs. Absolute Pressure

Suppose the pressure gauge on a full scuba tank reads 3000 psi, which is approximately 207 atmospheres. When the valve is opened, air begins to escape because the pressure inside the tank is greater than the atmospheric pressure outside the tank. Air continues to escape from the tank until the pressure inside the tank equals the pressure of the atmosphere outside the tank. At this point, the pressure gauge on the tank reads zero, even though the pressure inside the tank is actually 1 atmosphere—the same as the air pressure outside the tank.

Most pressure gauges, like the one on the scuba tank, are calibrated to read zero at atmospheric pressure. Pressure readings from such gauges are called **gauge pressure**, which is the pressure relative to the atmospheric pressure. When the pressure inside the tank is greater than atmospheric pressure, the gauge reports a positive value.

Some gauges are designed to measure negative pressure. For example, many physics experiments must take place in a vacuum chamber, a rigid chamber from which some of the air is pumped out. The pressure inside the vacuum chamber is less than atmospheric pressure, so the pressure gauge on the chamber reads a negative value.

Unlike gauge pressure, **absolute pressure** accounts for atmospheric pressure, which in effect adds to the pressure in any fluid not enclosed in a rigid container.

### Absolute Pressure

The absolute pressure, or total pressure, is the sum of gauge pressure and atmospheric pressure:

$$p_{\text{abs}} = p_{\text{g}} + p_{\text{atm}} \tag{14.11}$$

where $p_{\text{abs}}$ is absolute pressure, $p_{\text{g}}$ is gauge pressure, and $p_{\text{atm}}$ is atmospheric pressure.

For example, if a tire gauge reads 34 psi, then the absolute pressure is 34 psi plus 14.7 psi ( $p_{\text{atm}}$ in psi), or 48.7 psi (equivalent to 336 kPa).

In most cases, the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure in a fluid is zero (a negative absolute pressure is a pull). Thus, the smallest possible gauge pressure is $p_{\text{g}} = -p_{\text{atm}}$ (which makes $p_{\text{abs}}$ zero). There is no theoretical limit to how large a gauge pressure can be.

## Measuring Pressure

A host of devices are used for measuring pressure, ranging from tire gauges to blood pressure monitors. Many other types of pressure gauges are commonly used to test the pressure of fluids, such as mechanical pressure gauges. We will explore some of these in this section.

Any property that changes with pressure in a known way can be used to construct a pressure gauge. Some of the most common types include strain gauges, which use the change in the shape of a material with pressure; capacitance pressure gauges, which use the change in electric capacitance due to shape change with pressure; piezoelectric pressure gauges, which generate a voltage difference across a piezoelectric material under a pressure difference between the two sides; and ion gauges, which measure pressure by ionizing molecules in highly evacuated chambers. Different pressure gauges are useful in different pressure ranges and under different physical situations. Some examples are shown in Figure 14.11.

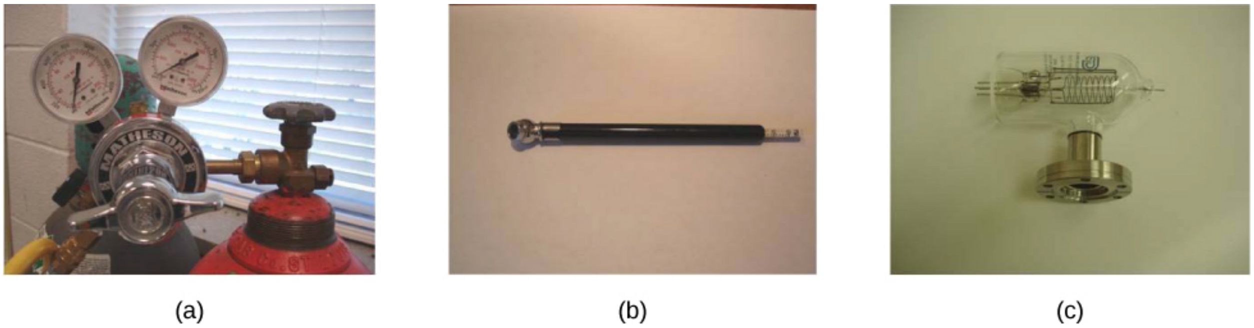



**Figure 14.11** (a) Gauges are used to measure and monitor pressure in gas cylinders. Compressed gases are used in many industrial as well as medical applications. (b) Tire pressure gauges come in many different models, but all are meant for the same purpose: to measure the internal pressure of the tire. This enables the driver to keep the tires inflated at optimal pressure for load weight and driving conditions. (c) An ionization gauge is a high-sensitivity device used to monitor the pressure of gases in an enclosed system. Neutral gas molecules are ionized by the release of electrons, and the current is translated into a pressure reading. Ionization gauges are commonly used in industrial applications that rely on vacuum systems.

## Manometers

One of the most important classes of pressure gauges applies the property that pressure due to the weight of a fluid of constant density is given by $p = h\rho g$. The U-shaped tube shown in **Figure 14.12** is an example of a *manometer*; in part (a), both sides of the tube are open to the atmosphere, allowing atmospheric pressure to push down on each side equally so that its effects cancel.

A manometer with only one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $p_g = h\rho g$ and is found by measuring *h*. For example, suppose one side of the U-tube is connected to some source of pressure $p_{abs}$, such as the balloon in part (b) of the figure or the vacuum-packed peanut jar shown in part (c). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In part (b), $p_{abs}$ is greater than atmospheric pressure, whereas in part (c), $p_{abs}$ is less than atmospheric pressure. In both cases, $p_{abs}$ differs from atmospheric pressure by an amount $h\rho g$, where $\rho$ is the density of the fluid in the manometer. In part (b), $p_{abs}$ can support a column of fluid of height *h*, so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure $p_g$ is positive). In part (c), atmospheric pressure can support a column of fluid of height *h*, so $p_{abs}$ is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure $p_g$ is negative).

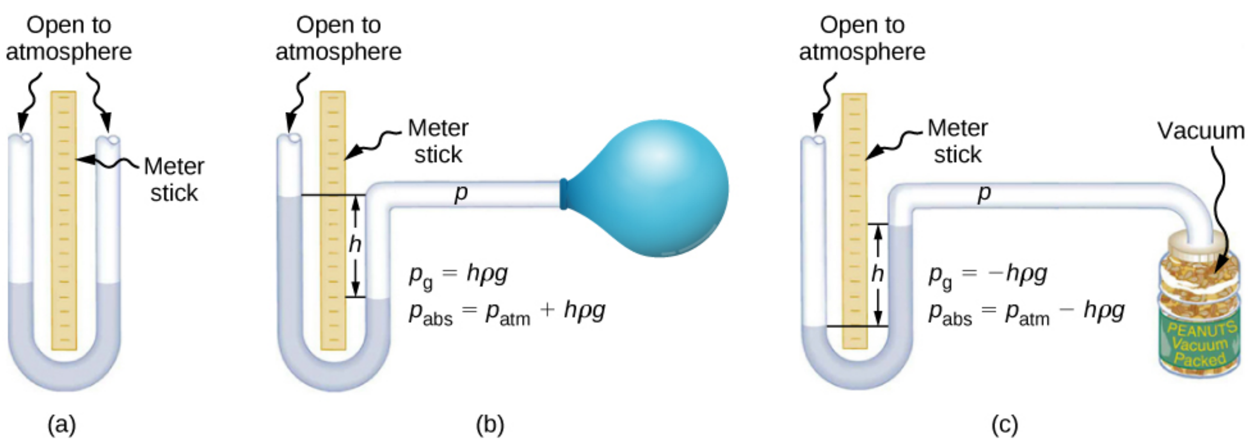



**Figure 14.12** An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and liquid will flow from the deeper side. (b) A positive gauge pressure $p_g = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height *h*. (c) Similarly, atmospheric pressure is greater than a negative gauge pressure $p_g$ by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

## Barometers

Manometers typically use a U-shaped tube of a fluid (often mercury) to measure pressure. A *barometer* (see **Figure 14.13**) is a device that typically uses a single column of mercury to measure atmospheric pressure. The barometer, invented by the Italian mathematician and physicist Evangelista Torricelli (1608–1647) in 1643, is constructed from a glass tube closed at one end and filled with mercury. The tube is then inverted and placed in a pool of mercury. This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = p_{\text{atm}}$. When atmospheric pressure varies, the mercury rises or falls.

Weather forecasters closely monitor changes in atmospheric pressure (often reported as barometric pressure), as rising mercury typically signals improving weather and falling mercury indicates deteriorating weather. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures.

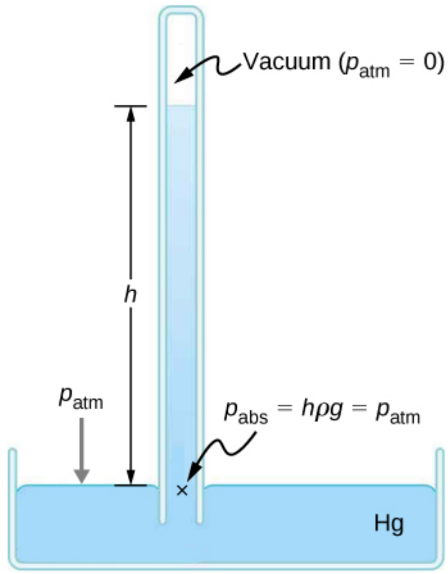



**Figure 14.13** A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height *h* because the pressure above the mercury is zero.

## Example 14.2

### Fluid Heights in an Open U-Tube

A U-tube with both ends open is filled with a liquid of density $\rho_1$ to a height *h* on both sides (**Figure 14.14**). A liquid of density $\rho_2 < \rho_1$ is poured into one side and Liquid 2 settles on top of Liquid 1. The heights on the two sides are different. The height to the top of Liquid 2 from the interface is $h_2$ and the height to the top of Liquid 1 from the level of the interface is $h_1$. Derive a formula for the height difference.

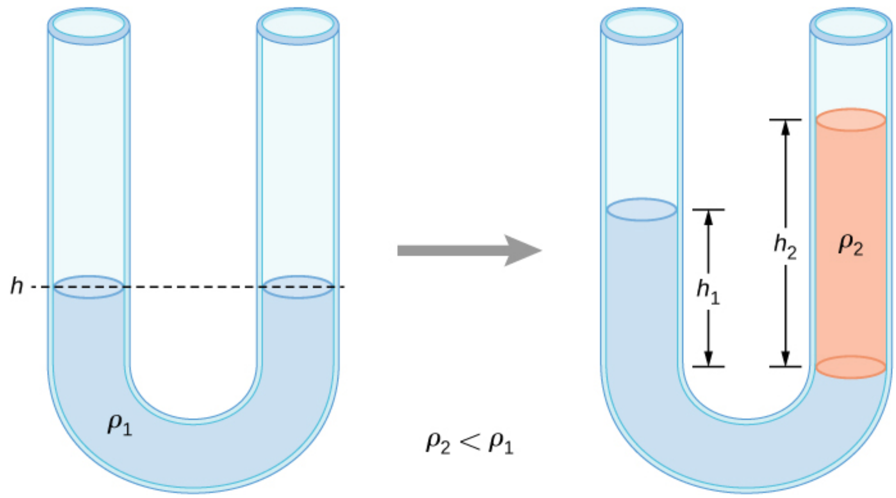



**Figure 14.14** Two liquids of different densities are shown in a U-tube.

### Strategy

The pressure at points at the same height on the two sides of a U-tube must be the same as long as the two points are in the same liquid. Therefore, we consider two points at the same level in the two arms of the tube: One point is the interface on the side of the Liquid 2 and the other is a point in the arm with Liquid 1 that is at the same level as the interface in the other arm. The pressure at each point is due to atmospheric pressure plus the weight of the liquid above it.

$$\text{Pressure on the side with Liquid 1} = p_0 + \rho_1 g h_1$$
$$\text{Pressure on the side with Liquid 2} = p_0 + \rho_2 g h_2$$

### Solution

Since the two points are in Liquid 1 and are at the same height, the pressure at the two points must be the same. Therefore, we have

$$p_0 + \rho_1 g h_1 = p_0 + \rho_2 g h_2.$$

Hence,

$$\rho_1 h_1 = \rho_2 h_2.$$

This means that the difference in heights on the two sides of the U-tube is

$$h_2 - h_1 = \left(1 - \frac{p_1}{p_2}\right)h_2.$$

The result makes sense if we set $p_2 = p_1$, which gives $h_2 = h_1$. If the two sides have the same density, they have the same height.

**14.2 Check Your Understanding** Mercury is a hazardous substance. Why do you suppose mercury is typically used in barometers instead of a safer fluid such as water?

## Units of pressure

As stated earlier, the SI unit for pressure is the pascal (Pa), where

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

In addition to the pascal, many other units for pressure are in common use (**Table 14.3**). In meteorology, atmospheric pressure is often described in the unit of millibars (mb), where

$$1000 \text{ mb} = 1 \times 10^5 \text{ Pa}.$$

The millibar is a convenient unit for meteorologists because the average atmospheric pressure at sea level on Earth is $1.013 \times 10^5 \text{ Pa} = 1013 \text{ mb} = 1 \text{ atm}$. Using the equations derived when considering pressure at a depth in a fluid, pressure can also be measured as millimeters or inches of mercury. The pressure at the bottom of a 760-mm column of mercury at $0\,°\text{C}$ in a container where the top part is evacuated is equal to the atmospheric pressure. Thus, 760 mm Hg is also used in place of 1 atmosphere of pressure. In vacuum physics labs, scientists often use another unit called the torr, named after Torricelli, who, as we have just seen, invented the mercury manometer for measuring pressure. One torr is equal to a pressure of 1 mm Hg.

| Unit | Definition |
|---|---|
| SI unit: the Pascal | $1 \text{ Pa} = 1 \text{ N/m}^2$ |
| English unit: pounds per square inch ( $\text{lb/in.}^2$ or psi) | $1 \text{ psi} = 7.015 \times 10^3 \text{ Pa}$ |
| Other units of pressure | $1 \text{ atm} = 760 \text{ mmHg} = 1.013 \times 10^5 \text{ Pa} = 14.7 \text{ psi} = 29.9 \text{ inches of Hg} = 1013 \text{ mb}$ |
| | $1 \text{ bar} = 10^5 \text{ Pa}$ |
| | $1 \text{ torr} = 1 \text{ mm Hg} = 122.39 \text{ Pa}$ |

**Table 14.3 Summary of the Units of Pressure**

# 14.3 | Pascal's Principle and Hydraulics

## Learning Objectives

By the end of this section, you will be able to:

- State Pascal’s principle
- Describe applications of Pascal’s principle
- Derive relationships between forces in a hydraulic system

In 1653, the French philosopher and scientist Blaise Pascal published his *Treatise on the Equilibrium of Liquids*, in which he discussed principles of static fluids. A static fluid is a fluid that is not in motion. When a fluid is not flowing, we say that the fluid is in static equilibrium. If the fluid is water, we say it is in **hydrostatic equilibrium**. For a fluid in static equilibrium, the net force on any part of the fluid must be zero; otherwise the fluid will start to flow.

Pascal’s observations—since proven experimentally—provide the foundation for hydraulics, one of the most important developments in modern mechanical technology. Pascal observed that a change in pressure applied to an enclosed fluid is transmitted undiminished throughout the fluid and to the walls of its container. Because of this, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal’s principle implies that the total pressure in a fluid is the sum of the pressures from different sources. A good example is the fluid at a depth depends on the depth of the fluid and the pressure of the atmosphere.

## Pascal’s Principle

**Pascal’s principle** (also known as Pascal’s law) states that when a change in pressure is applied to an enclosed fluid, it is transmitted undiminished to all portions of the fluid and to the walls of its container. In an enclosed fluid, since atoms of the fluid are free to move about, they transmit pressure to all parts of the fluid *and* to the walls of the container. Any change in pressure is transmitted undiminished.

Note that this principle does not say that the pressure is the same at all points of a fluid—which is not true, since the pressure in a fluid near Earth varies with height. Rather, this principle applies to the *change* in pressure. Suppose you place some water in a cylindrical container of height *H* and cross-sectional area *A* that has a movable piston of mass *m* (**Figure 14.15**). Adding weight *Mg* at the top of the piston increases the pressure at the top by *Mg*/*A*, since the additional weight also acts over area *A* of the lid:

$$\Delta p_{\text{top}} = \frac{Mg}{A}.$$

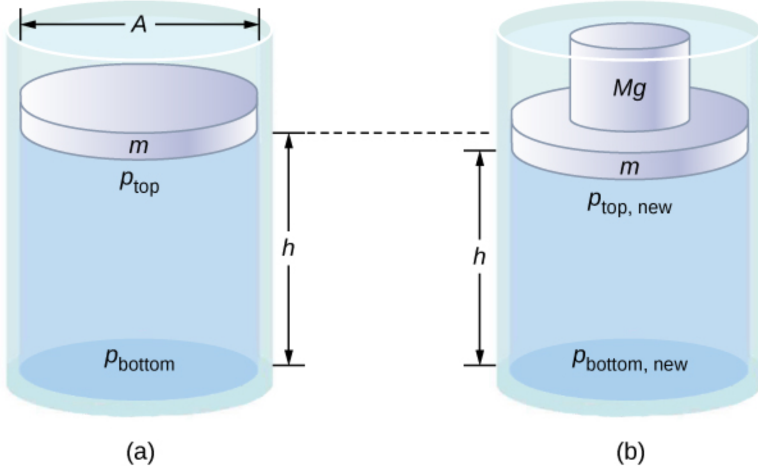



**Figure 14.15** Pressure in a fluid changes when the fluid is compressed. (a) The pressure at the top layer of the fluid is different from pressure at the bottom layer. (b) The increase in pressure by adding weight to the piston is the same everywhere, for example, $p_{\text{top new}} - p_{\text{top}} = p_{\text{bottom new}} - p_{\text{bottom}}$.

According to Pascal's principle, the pressure at all points in the water changes by the same amount, *Mg*/*A*. Thus, the pressure at the bottom also increases by *Mg*/*A*. The pressure at the bottom of the container is equal to the sum of the atmospheric pressure, the pressure due the fluid, and the pressure supplied by the mass. The change in pressure at the bottom of the container due to the mass is

$$\Delta p_{\text{bottom}} = \frac{Mg}{A}.$$

Since the pressure changes are the same everywhere in the fluid, we no longer need subscripts to designate the pressure change for top or bottom:

$$\Delta p = \Delta p_{\text{top}} = \Delta p_{\text{bottom}} = \Delta p_{\text{everywhere}}.$$

Pascal's Barrel is a great demonstration of Pascal's principle. Watch a **simulation (https://openstaxcollege.org/l/21pascalbarrel)** of Pascal's 1646 experiment, in which he demonstrated the effects of changing pressure in a fluid.

## Applications of Pascal's Principle and Hydraulic Systems

Hydraulic systems are used to operate automotive brakes, hydraulic jacks, and numerous other mechanical systems (**Figure 14.16**).

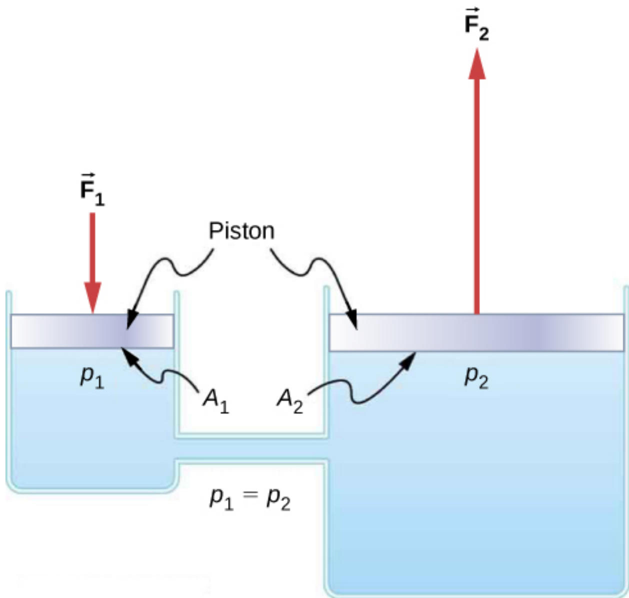



**Figure 14.16** A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force $\vec{F}_1$ on the left piston creates a change in pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force $\vec{F}_2$ on the right piston that is larger than $\vec{F}_1$ because the right piston has a larger surface area.

We can derive a relationship between the forces in this simple hydraulic system by applying Pascal's principle. Note first that the two pistons in the system are at the same height, so there is no difference in pressure due to a difference in depth. The pressure due to $F_1$ acting on area $A_1$ is simply

$$p_1 = \frac{F_1}{A_1}, \text{ as defined y } p = \frac{F}{A}.$$

According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure $p_2$ is felt at the other piston that is equal to $p_1$. That is, $p_1 = p_2$. However, since $p_2 = F_2/A_2$, we see that

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}. \tag{14.12}$$

This equation relates the ratios of force to area in any hydraulic system, provided that the pistons are at the same vertical height and that friction in the system is negligible.

Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in **Figure 14.16** and the right cylinder has an area five times greater, then the output force is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

The **hydraulic jack** is such a hydraulic system. A hydraulic jack is used to lift heavy loads, such as the ones used by auto mechanics to raise an automobile. It consists of an incompressible fluid in a U-tube fitted with a movable piston on each side. One side of the U-tube is narrower than the other. A small force applied over a small area can balance a much larger force on the other side over a larger area (**Figure 14.17**).

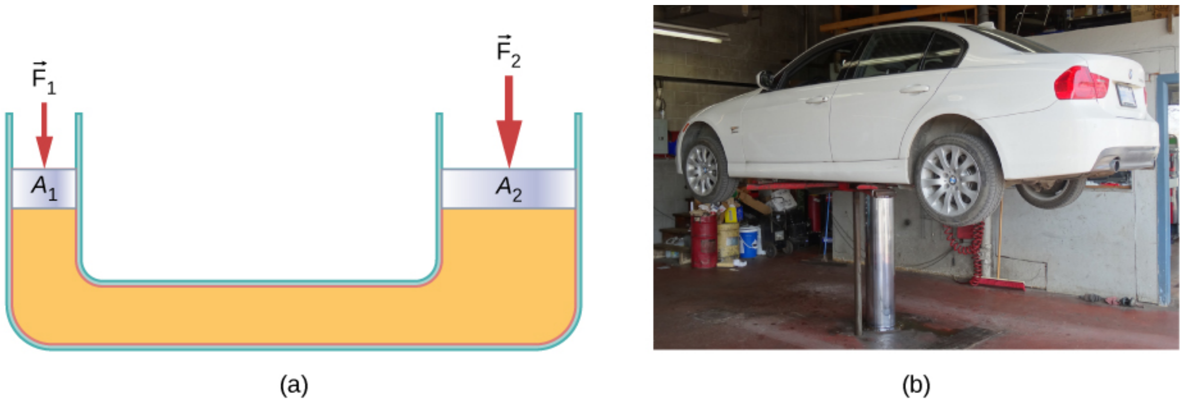



**Figure 14.17** (a) A hydraulic jack operates by applying forces $(F_1, F_2)$ to an incompressible fluid in a U-tube, using a movable piston $(A_1, A_2)$ on each side of the tube. (b) Hydraulic jacks are commonly used by car mechanics to lift vehicles so that repairs and maintenance can be performed.

From Pascal's principle, it can be shown that the force needed to lift the car is less than the weight of the car:

$$F_1 = \frac{A_1}{A_2} F_2,$$

where $F_1$ is the force applied to lift the car, $A_1$ is the cross-sectional area of the smaller piston, $A_2$ is the cross sectional area of the larger piston, and $F_2$ is the weight of the car.

## Example 14.3

### Calculating Force on Wheel Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in Figure 14.18. Suppose a force of 100 N is applied to the brake pedal, which acts on the pedal cylinder (acting as a "master" cylinder) through a lever. A force of 500 N is exerted on the pedal cylinder. Pressure created in the pedal cylinder is transmitted to the four wheel cylinders. The pedal cylinder has a diameter of 0.500 cm and each wheel cylinder has a diameter of 2.50 cm. Calculate the magnitude of the force $F_2$ created at each of the wheel cylinders.

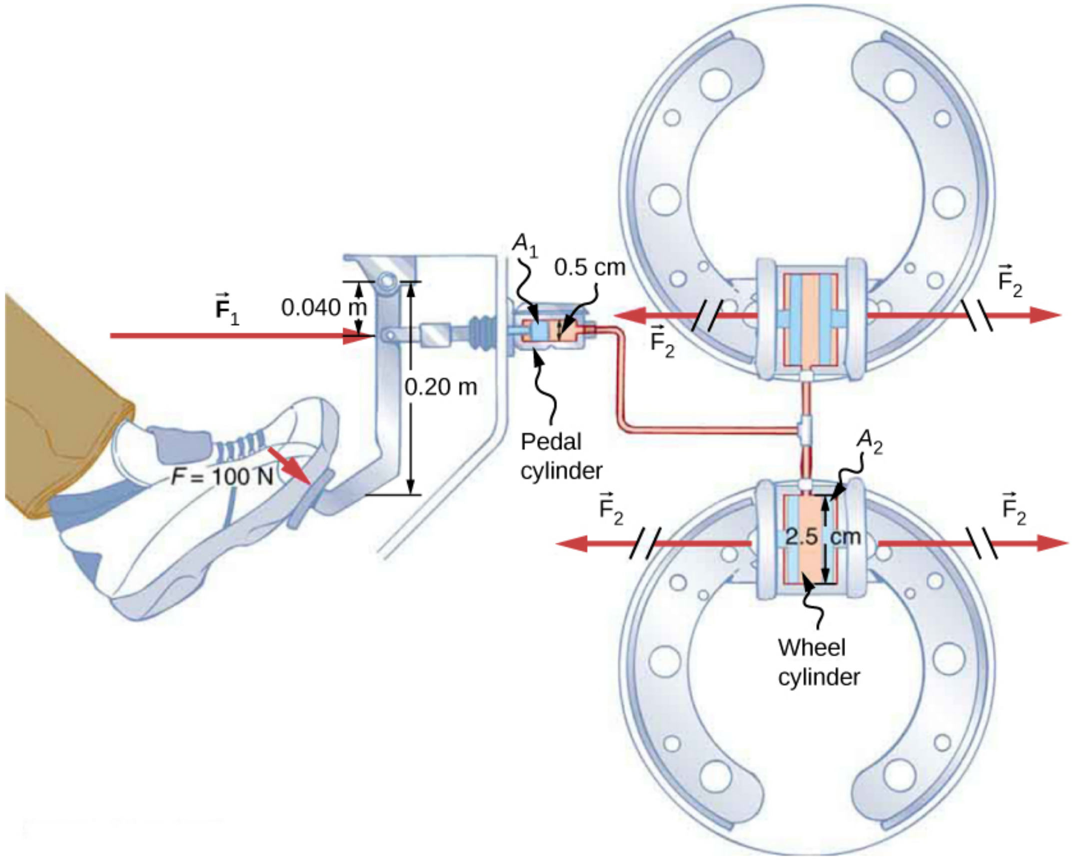



**Figure 14.18** Hydraulic brakes use Pascal's principle. The driver pushes the brake pedal, exerting a force that is increased by the simple lever and again by the hydraulic system. Each of the identical wheel cylinders receives the same pressure and, therefore, creates the same force output $F_2$. The circular cross-sectional areas of the pedal and wheel cylinders are represented by $A_1$ and $A_2$, respectively.

### Strategy

We are given the force $F_1$ applied to the pedal cylinder. The cross-sectional areas $A_1$ and $A_2$ can be calculated from their given diameters. Then we can use the following relationship to find the force $F_2$:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

Manipulate this algebraically to get $F_2$ on one side and substitute known values.

### Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

$$\begin{aligned} F_2 &= \frac{A_2}{A_1}F_1 = \frac{\pi r_2^2}{\pi r_1^2}F_1 \\ &= \frac{(1.25\text{ cm})^2}{(0.250\text{ cm})^2} \times 500\text{ N} = 1.25 \times 10^4\text{ N}. \end{aligned}$$

### Significance

This value is the force exerted by each of the four wheel cylinders. Note that we can add as many wheel cylinders as we wish. If each has a 2.50-cm diameter, each will exert $1.25 \times 10^4$ N. A simple hydraulic system, as an example of a simple machine, can increase force but cannot do more work than is done on it. Work is force times distance moved, and the wheel cylinder moves through a smaller distance than the pedal cylinder. Furthermore, the more wheels added, the smaller the distance each one moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system.

**14.3 Check Your Understanding** Would a hydraulic press still operate properly if a gas is used instead of a liquid?

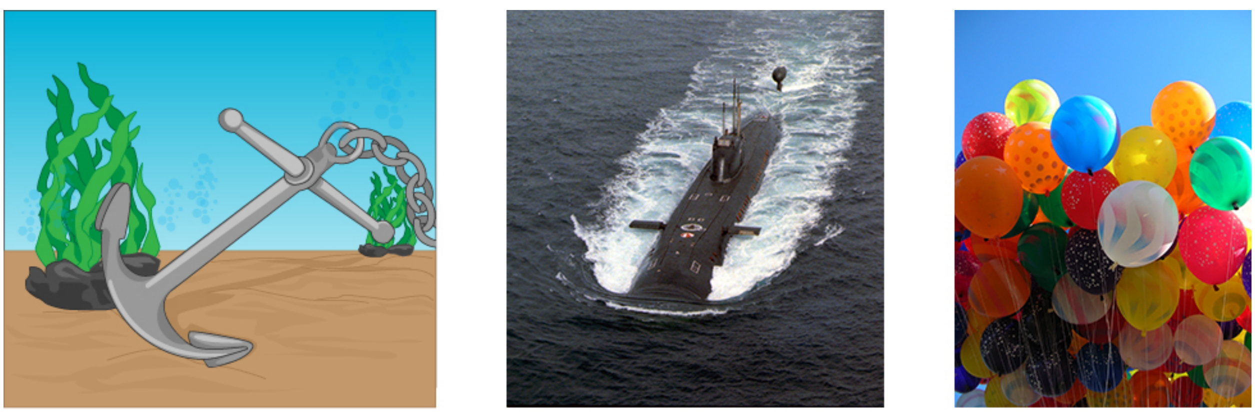
# 14.4 | Archimedes' Principle and Buoyancy

## Learning Objectives

By the end of this section, you will be able to:

- Define buoyant force
- State Archimedes' principle
- Describe the relationship between density and Archimedes' principle

When placed in a fluid, some objects float due to a buoyant force. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected (**Figure 14.19**)?



(a) (b) (c)

**Figure 14.19** (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by "Crystl"/Flickr)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on top of the object. There is an upward force, or **buoyant force**, on any object in any fluid (**Figure 14.20**). If the buoyant force is greater than the object's weight, the object rises to the surface and floats. If the buoyant force is less than the object's weight, the object sinks. If the buoyant force equals the object's weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.

### Buoyant Force

The buoyant force is the upward force on any object in any fluid.

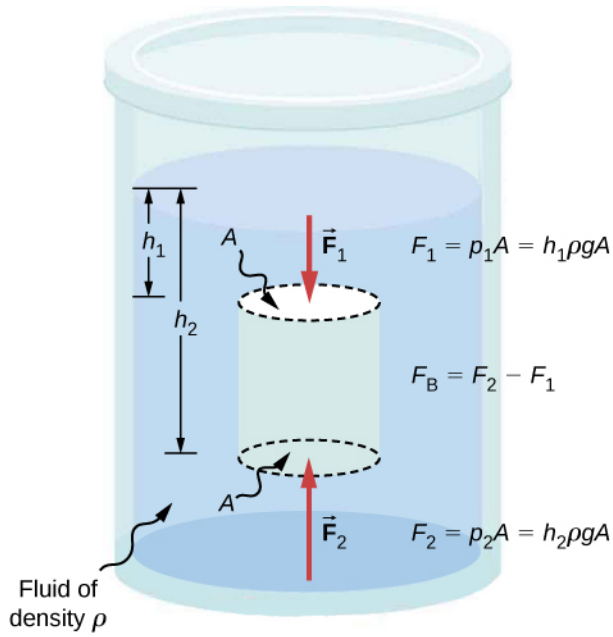



**Figure 14.20** Pressure due to the weight of a fluid increases with depth because $p = hpg$ . This change in pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. The differences in the force results in the buoyant force $F_B$ . (Horizontal forces cancel.)

## Archimedes' Principle

Just how large a force is buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in **Figure 14.21**. If the object were not in the fluid, the space the object occupied would be filled by fluid having a weight $w_{fl}$. This weight is supported by the surrounding fluid, so the buoyant force must equal $w_{fl}$, the weight of the fluid displaced by the object.

### Archimedes' Principle

The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes' principle** is

$$F_B = w_{fl},$$

where $F_B$ is the buoyant force and $w_{fl}$ is the weight of the fluid displaced by the object.

This principle is named after the Greek mathematician and inventor Archimedes (ca. 287–212 BCE), who stated this principle long before concepts of force were well established.

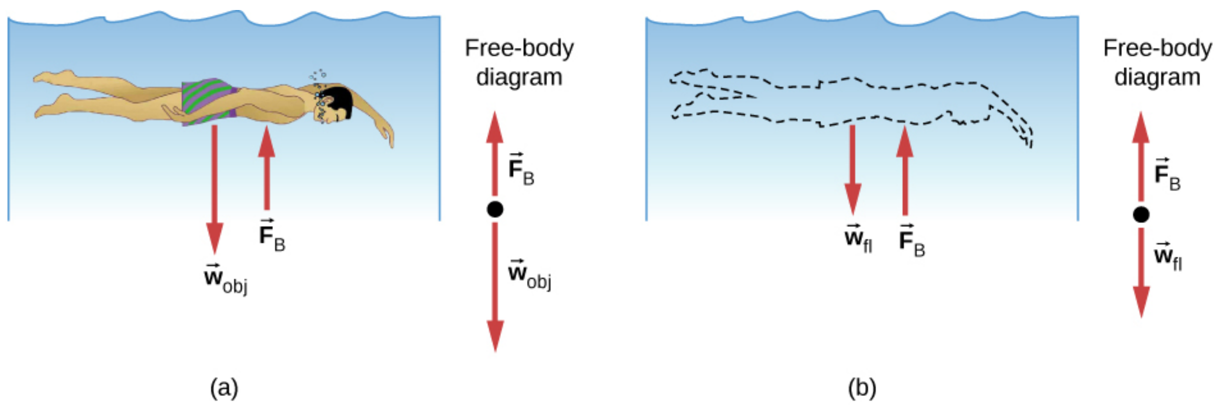



**Figure 14.21** (a) An object submerged in a fluid experiences a buoyant force $F_B$. If $F_B$ is greater than the weight of the object, the object rises. If $F_B$ is less than the weight of the object, the object sinks. (b) If the object is removed, it is replaced by fluid having weight $w_{fl}$. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced.

Archimedes' principle refers to the force of buoyancy that results when a body is submerged in a fluid, whether partially or wholly. The force that provides the pressure of a fluid acts on a body perpendicular to the surface of the body. In other words, the force due to the pressure at the bottom is pointed up, while at the top, the force due to the pressure is pointed down; the forces due to the pressures at the sides are pointing into the body.

Since the bottom of the body is at a greater depth than the top of the body, the pressure at the lower part of the body is higher than the pressure at the upper part, as shown in **Figure 14.20**. Therefore a net upward force acts on the body. This upward force is the force of buoyancy, or simply *buoyancy*.

The exclamation "Eureka" (meaning "I found it") has been credited to Archimedes as he made the discovery that would lead to Archimedes' principle. Some say it all started in a bathtub. To read the story, visit **NASA (https://openstaxcollege.org/l/21archNASA)** or explore **Scientific American (https://openstaxcollege.org/l/21archsciamer)** to learn more.

## Density and Archimedes' Principle

If you drop a lump of clay in water, it will sink. But if you mold the same lump of clay into the shape of a boat, it will float. Because of its shape, the clay boat displaces more water than the lump and experiences a greater buoyant force, even though its mass is the same. The same is true of steel ships.

The average density of an object is what ultimately determines whether it floats. If an object's average density is less than that of the surrounding fluid, it will float. The reason is that the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density compares to the density of the fluid. In **Figure 14.22**, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship when loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call $V_{fl}$. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\rho_{\text{obj}}},$$

where $\rho_{\text{obj}}$ is the average density of the object and $\rho_{\text{fl}}$ is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.$$

We can use this relationship to measure densities.

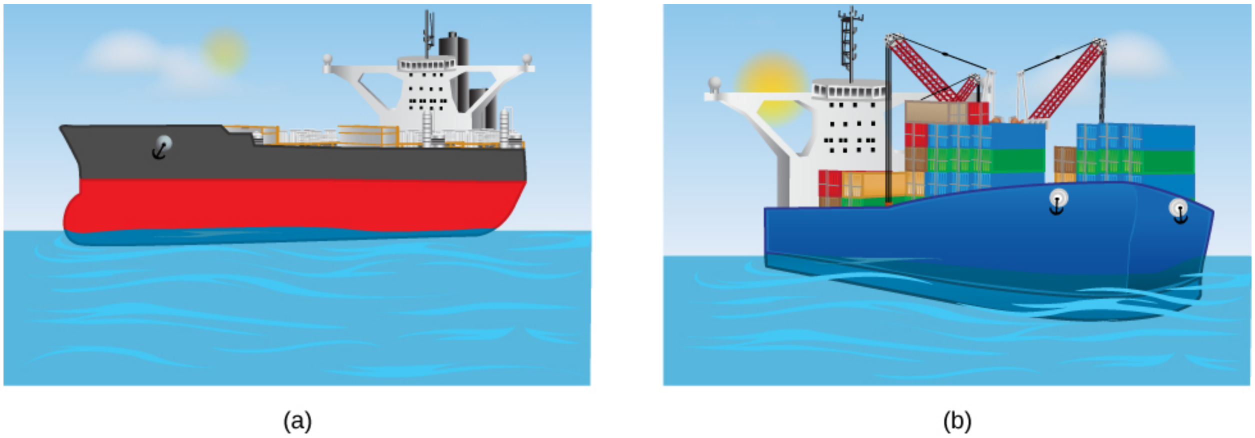



**Figure 14.22** An unloaded ship (a) floats higher in the water than a loaded ship (b).

## Example 14.4

### Calculating Average Density

Suppose a 60.0-kg woman floats in fresh water with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

### Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

$$\rho_{\text{obj}} = \rho_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, so we can calculate the woman's density.

### Solution

Entering the known values into the expression for her density, we obtain

$$\rho_{\text{person}} = 0.970 \cdot \left(10^3 \, \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

### Significance

The woman's density is less than the fluid density. We expect this because she floats.

Numerous lower-density objects or substances float in higher-density fluids: oil on water, a hot-air balloon in the atmosphere, a bit of cork in wine, an iceberg in salt water, and hot wax in a "lava lamp," to name a few. A less obvious example is mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

# Measuring Density

One of the most common techniques for determining density is shown in **Figure 14.23**.

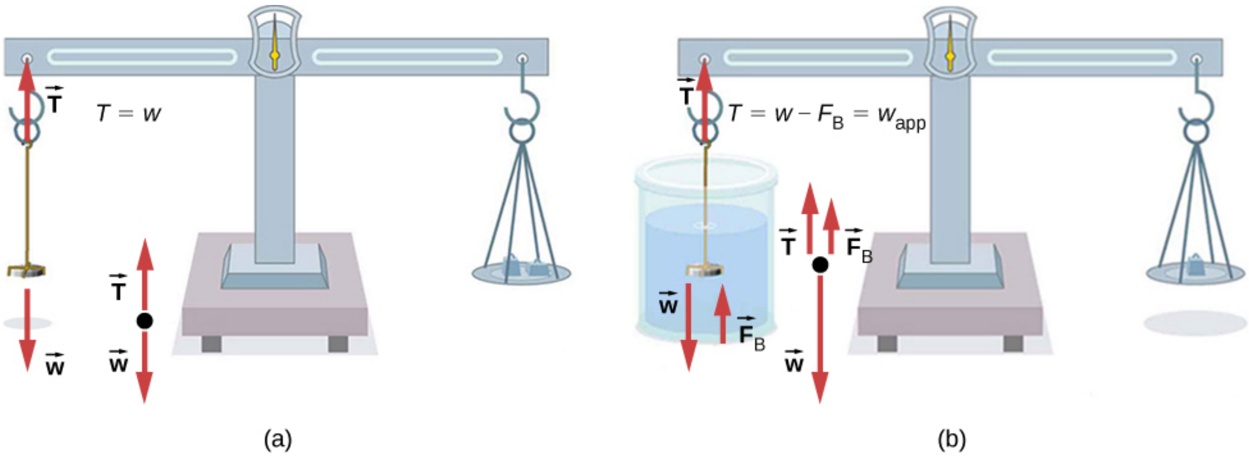



**Figure 14.23** (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. We can use this same technique to determine the density of the fluid if the density of the coin is known.

All of these calculations are based on Archimedes' principle, which states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object appears to weigh less when submerged; we call this measurement the object's apparent weight. The object suffers an apparent weight loss equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an apparent mass loss equal to the mass of fluid displaced. That is, apparent weight loss equals weight of fluid displaced, or apparent mass loss equals mass of fluid displaced.

# 14.5 | Fluid Dynamics

## Learning Objectives

By the end of this section, you will be able to:

- Describe the characteristics of flow
- Calculate flow rate
- Describe the relationship between flow rate and velocity
- Explain the consequences of the equation of continuity to the conservation of mass

The first part of this chapter dealt with fluid statics, the study of fluids at rest. The rest of this chapter deals with fluid dynamics, the study of fluids in motion. Even the most basic forms of fluid motion can be quite complex. For this reason, we limit our investigation to **ideal fluids** in many of the examples. An ideal fluid is a fluid with negligible **viscosity**. Viscosity is a measure of the internal friction in a fluid; we examine it in more detail in Viscosity and Turbulence. In a few examples, we examine an incompressible fluid—one for which an extremely large force is required to change the volume—since the density in an incompressible fluid is constant throughout.

## Characteristics of Flow

Velocity vectors are often used to illustrate fluid motion in applications like meteorology. For example, wind—the fluid motion of air in the atmosphere—can be represented by vectors indicating the speed and direction of the wind at any given point on a map. Figure 14.24 shows velocity vectors describing the winds during Hurricane Arthur in 2014.

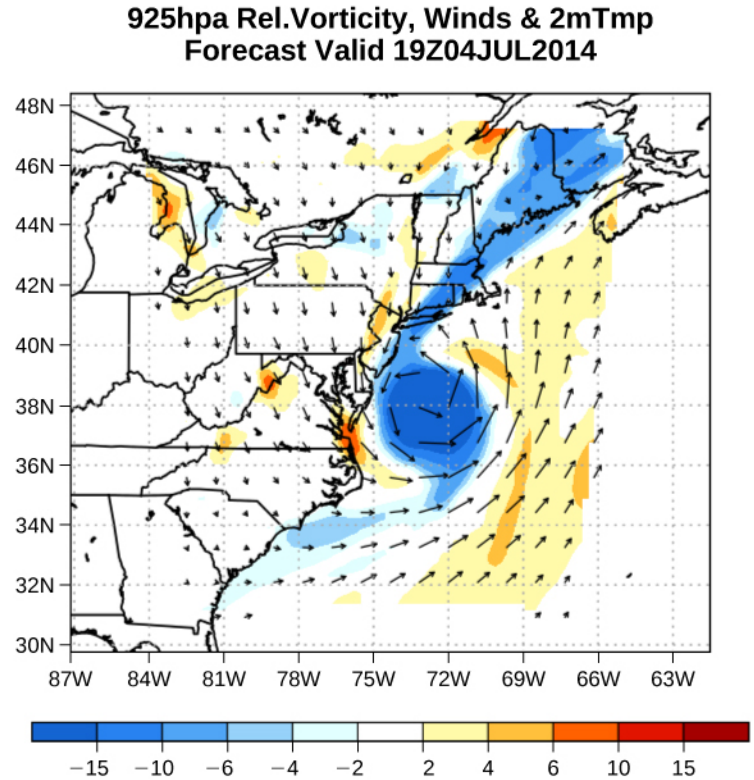



**Figure 14.24** The velocity vectors show the flow of wind in Hurricane Arthur. Notice the circulation of the wind around the eye of the hurricane. Wind speeds are highest near the eye. The colors represent the relative vorticity, a measure of turning or spinning of the air.

Another method for representing fluid motion is a *streamline*. A streamline represents the path of a small volume of fluid as it flows. The velocity is always tangential to the streamline. The diagrams in **Figure 14.25** use streamlines to illustrate two examples of fluids moving through a pipe. The first fluid exhibits a **laminar flow** (sometimes described as a steady flow), represented by smooth, parallel streamlines. Note that in the example shown in part (a), the velocity of the fluid is greatest in the center and decreases near the walls of the pipe due to the viscosity of the fluid and friction between the pipe walls and the fluid. This is a special case of laminar flow, where the friction between the pipe and the fluid is high, known as no slip boundary conditions. The second diagram represents **turbulent flow**, in which streamlines are irregular and change over time. In turbulent flow, the paths of the fluid flow are irregular as different parts of the fluid mix together or form small circular regions that resemble whirlpools. This can occur when the speed of the fluid reaches a certain critical speed.

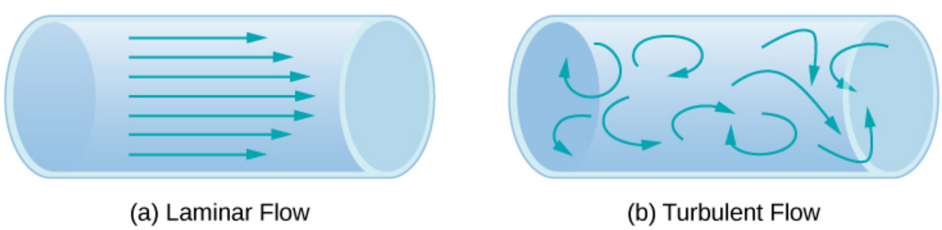



**Figure 14.25** (a) Laminar flow can be thought of as layers of fluid moving in parallel, regular paths. (b) In turbulent flow, regions of fluid move in irregular, colliding paths, resulting in mixing and swirling.

## Flow Rate and its Relation to Velocity

The volume of fluid passing by a given location through an area during a period of time is called **flow rate** *Q*, or more precisely, volume flow rate. In symbols, this is written as

$$Q = \frac{dV}{dt} \tag{14.13}$$

where $V$ is the volume and $t$ is the elapsed time. In **Figure 14.26**, the volume of the cylinder is $Ax$, so the flow rate is

$$Q = \frac{dV}{dt} = \frac{d}{dt}(Ax) = A\frac{dx}{dt} = Av.$$

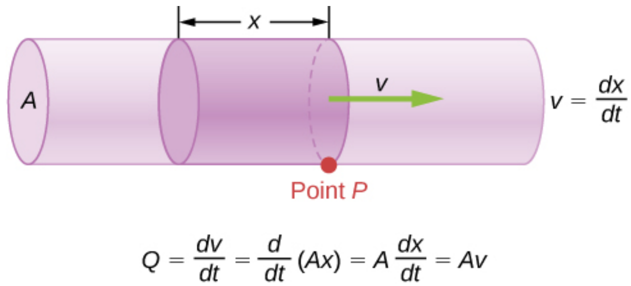



**Figure 14.26** Flow rate is the volume of fluid flowing past a point through the area $A$ per unit time. Here, the shaded cylinder of fluid flows past point $P$ in a uniform pipe in time $t$.

The SI unit for flow rate is $\text{m}^3/\text{s}$, but several other units for $Q$ are in common use, such as liters per minute (L/min). Note that a liter (L) is 1/1000 of a cubic meter or 1000 cubic centimeters $(10^{-3}\ \text{m}^3$ or $10^3\ \text{cm}^3)$.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, consider the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size and shape of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. **Figure 14.26** illustrates the volume flow rate. The volume flow rate is $Q = \frac{dV}{dt} = Av$, where $A$ is the cross-sectional area of the pipe and $v$ is the magnitude of the velocity.

The precise relationship between flow rate $Q$ and average speed $v$ is

$$Q = Av,$$

where $A$ is the cross-sectional area and $v$ is the average speed. The relationship tells us that flow rate is directly proportional to both the average speed of the fluid and the cross-sectional area of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. **Figure 14.26** illustrates how this relationship is obtained. The shaded cylinder has a volume $V = Ad$, which flows past the point $P$ in a time $t$. Dividing both sides of this relationship by $t$ gives

$$\frac{V}{t} = \frac{Ad}{t}.$$

We note that $Q = V/t$ and the average speed is $v = d/t$. Thus the equation becomes $Q = Av$.

**Figure 14.27** shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. The flow is continuous because they are no sources or sinks that add or remove mass, so the mass flowing into the pipe must be equal the mass flowing out of the pipe. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for arbitrary points 1 and 2,

$$\begin{aligned} Q_1 &= Q_2, \\ A_1 v_1 &= A_2 v_2. \end{aligned} \tag{14.14}$$

This is called the *equation of continuity* and is valid for any incompressible fluid (with constant density). The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: It emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

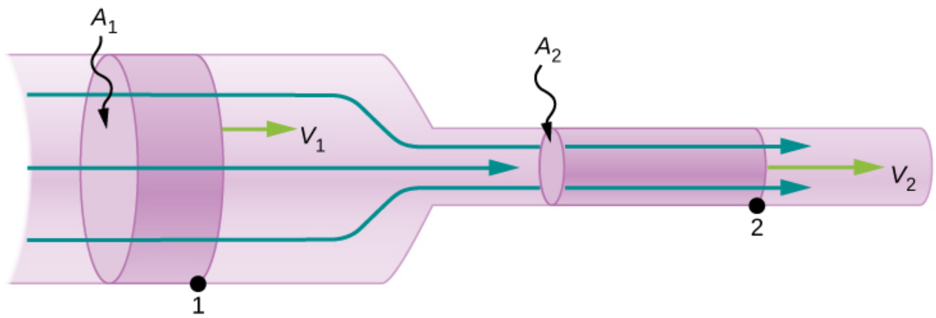



**Figure 14.27** When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed decreases when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, so the equation must be applied with caution to gases if they are subjected to compression or expansion.

## Example 14.5

### Calculating Fluid Speed through a Nozzle

A nozzle with a diameter of 0.500 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

### Strategy

We can use the relationship between flow rate and speed to find both speeds. We use the subscript 1 for the hose and 2 for the nozzle.

### Solution

a. We solve the flow rate equation for speed and use $\pi r_1{}^2$ for the cross-sectional area of the hose, obtaining

$$v = \frac{Q}{A} = \frac{Q}{\pi r_1^2}.$$

Substituting values and using appropriate unit conversions yields

$$v = \frac{(0.500\ \text{L/s})(10^{-3}\ \text{m}^3/\text{L})}{3.14(9.00 \times 10^{-3}\ \text{m})^2} = 1.96\ \text{m/s}.$$

b. We could repeat this calculation to find the speed in the nozzle $v_2$, but we use the equation of continuity to give a somewhat different insight. The equation states

$$A_1 v_1 = A_2 v_2.$$

Solving for $v_2$ and substituting $\pi r^2$ for the cross-sectional area yields

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_1^2}{r_2^2} v_1.$$

Substituting known values,

$$v_2 = \frac{(0.900\text{ cm})^2}{(0.250\text{ cm})^2} 1.96\text{ m/s} = 25.5\text{ m/s}.$$

### Significance

A speed of 1.96 m/s is about right for water emerging from a hose with no nozzle. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the square of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

## Mass Conservation

The rate of flow of a fluid can also be described by the *mass flow rate* or mass rate of flow. This is the rate at which a mass of the fluid moves past a point. Refer once again to **Figure 14.26**, but this time consider the mass in the shaded volume. The mass can be determined from the density and the volume:

$$m = \rho V = \rho A x.$$

The mass flow rate is then

$$\frac{dm}{dt} = \frac{d}{dt}(\rho A x) = \rho A \frac{dx}{dt} = \rho A v,$$

where $\rho$ is the density, $A$ is the cross-sectional area, and $v$ is the magnitude of the velocity. The mass flow rate is an important quantity in fluid dynamics and can be used to solve many problems. Consider **Figure 14.28**. The pipe in the figure starts at the inlet with a cross sectional area of $A_1$ and constricts to an outlet with a smaller cross sectional area of $A_2$. The mass of fluid entering the pipe has to be equal to the mass of fluid leaving the pipe. For this reason the velocity at the outlet $(v_2)$ is greater than the velocity of the inlet $(v_1)$. Using the fact that the mass of fluid entering the pipe must be equal to the mass of fluid exiting the pipe, we can find a relationship between the velocity and the cross-sectional area by taking the rate of change of the mass in and the mass out:

$$\begin{aligned} \left(\frac{dm}{dt}\right)_1 &= \left(\frac{dm}{dt}\right)_2 \\ \rho_1 A_1 v_1 &= \rho_2 A_2 v_2. \end{aligned} \tag{14.15}$$

**Equation 14.15** is also known as the continuity equation in general form. If the density of the fluid remains constant through the constriction—that is, the fluid is incompressible—then the density cancels from the continuity equation,

$$A_1 v_1 = A_2 v_2.$$

The equation reduces to show that the volume flow rate into the pipe equals the volume flow rate out of the pipe.

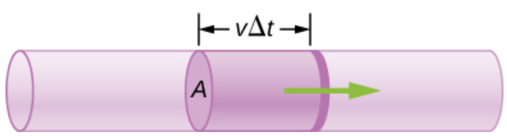



**Figure 14.28** Geometry for deriving the equation of continuity. The amount of liquid entering the cross-sectional (shaded) area must equal the amount of liquid leaving the cross-sectional area if the liquid is incompressible.

# 14.6 | Bernoulli's Equation

## Learning Objectives

By the end of this section, you will be able to:

- Explain the terms in Bernoulli's equation
- Explain how Bernoulli's equation is related to the conservation of energy
- Describe how to derive Bernoulli's principle from Bernoulli's equation
- Perform calculations using Bernoulli's principle
- Describe some applications of Bernoulli's principle

As we showed in **Figure 14.27**, when a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. The increased kinetic energy comes from the net work done on the fluid to push it into the channel. Also, if the fluid changes vertical position, work is done on the fluid by the gravitational force.

A pressure difference occurs when the channel narrows. This pressure difference results in a net force on the fluid because the pressure times the area equals the force, and this net force does work. Recall the work-energy theorem,

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

The net work done increases the fluid's kinetic energy. As a result, the pressure drops in a rapidly moving fluid whether or not the fluid is confined to a tube.

There are many common examples of pressure dropping in rapidly moving fluids. For instance, shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The reason is that the high-velocity stream of water and air creates a region of lower pressure inside the shower, whereas the pressure on the other side remains at the standard atmospheric pressure. This pressure difference results in a net force, pushing the curtain inward. Similarly, when a car passes a truck on the highway, the two vehicles seem to pull toward each other. The reason is the same: The high velocity of the air between the car and the truck creates a region of lower pressure between the vehicles, and they are pushed together by greater pressure on the outside (**Figure 14.29**). This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.

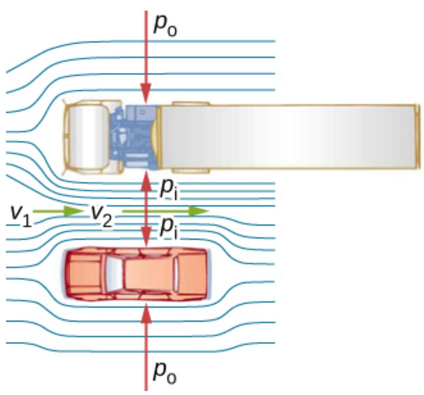



**Figure 14.29** An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed ( $v_2$ is greater than $v_1$ ), causing the pressure between them to drop ( $p_i$ is less than $p_o$). Greater pressure on the outside pushes the car and truck together.

## Energy Conservation and Bernoulli's Equation

The application of the principle of conservation of energy to frictionless laminar flow leads to a very useful relation between pressure and flow speed in a fluid. This relation is called **Bernoulli's equation**, named after Daniel Bernoulli (1700–1782), who published his studies on fluid motion in his book *Hydrodynamica* (1738).

Consider an incompressible fluid flowing through a pipe that has a varying diameter and height, as shown in **Figure 14.30**. Subscripts 1 and 2 in the figure denote two locations along the pipe and illustrate the relationships between the areas of the cross sections *A*, the speed of flow *v*, the height from ground *y*, and the pressure *p* at each point. We assume here that the density at the two points is the same—therefore, density is denoted by $\rho$ without any subscripts—and since the fluid in incompressible, the shaded volumes must be equal.

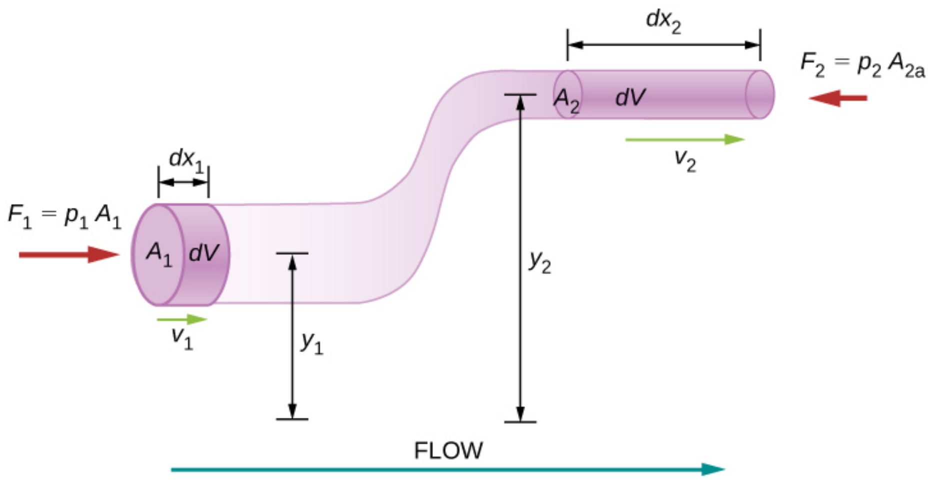



**Figure 14.30** The geometry used for the derivation of Bernoulli's equation.

We also assume that there are no viscous forces in the fluid, so the energy of any part of the fluid will be conserved. To derive Bernoulli's equation, we first calculate the work that was done on the fluid:

$$dW = F_1 dx_1 - F_2 dx_2$$
$$dW = p_1 A_1 dx_1 - p_2 A_2 dx_2 = p_1 dV - p_2 dV = (p_1 - p_2)dV.$$

The work done was due to the conservative force of gravity and the change in the kinetic energy of the fluid. The change in the kinetic energy of the fluid is equal to

$$dK = \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 v_1^2 = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right).$$

The change in potential energy is

$$dU = mgy_2 - mgy_1 = \rho dVg(y_2 - y_1).$$

The energy equation then becomes

$$\begin{aligned} dW &= dK + dU \\ (p_1 - p_2)dV &= \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) + \rho dVg(y_2 - y_1) \\ (p_1 - p_2) &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g(y_2 - y_1). \end{aligned}$$

Rearranging the equation gives Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

This relation states that the mechanical energy of any part of the fluid changes as a result of the work done by the fluid external to that part, due to varying pressure along the way. Since the two points were chosen arbitrarily, we can write Bernoulli's equation more generally as a conservation principle along the flow.

### Bernoulli's Equation

For an incompressible, frictionless fluid, the combination of pressure and the sum of kinetic and potential energy densities is constant not only over time, but also along a streamline:

$$p+\frac{1}{2}\rho v^2+\rho gy=\text{constant} \tag{14.16}$$

A special note must be made here of the fact that in a dynamic situation, the pressures at the same height in different parts of the fluid may be different if they have different speeds of flow.

## Analyzing Bernoulli's Equation

According to Bernoulli's equation, if we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, let us consider some specific situations that simplify and illustrate its use and meaning.

### Bernoulli's equation for static fluids

First consider the very simple situation where the fluid is static—that is, $v_1=v_2=0.$ Bernoulli's equation in that case is

$$p_1+\rho gh_1=p_2+\rho gh_2.$$

We can further simplify the equation by setting $h_2=0.$ (Any height can be chosen for a reference height of zero, as is often done for other situations involving gravitational force, making all other heights relative.) In this case, we get

$$p_2=p_1+\rho gh_1.$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by $h_1$, and consequently, $p_2$ is greater than $p_1$ by an amount $\rho gh_1$. In the very simplest case, $p_1$ is zero at the top of the fluid, and we get the familiar relationship $p=\rho gh$. (Recall that $p=\rho gh$ and $\Delta U_{\text{g}}=-mgh.$) Thus, Bernoulli's equation confirms the fact that the pressure change due to the weight of a fluid is $\rho gh$. Although we introduce Bernoulli's equation for fluid motion, it includes much of what we studied for static fluids earlier.

### Bernoulli's principle

Suppose a fluid is moving but its depth is constant—that is, $h_1=h_2$. Under this condition, Bernoulli's equation becomes

$$p_1+\frac{1}{2}\rho v_1^2=p_2+\frac{1}{2}\rho v_2^2.$$

Situations in which fluid flows at a constant depth are so common that this equation is often also called **Bernoulli's principle**, which is simply Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) Bernoulli's principle reinforces the fact that pressure drops as speed increases in a moving fluid: If $v_2$ is greater than $v_1$ in the equation, then $p_2$ must be less than $p_1$ for the equality to hold.

## Example 14.6

### Calculating Pressure

In Example 14.5, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01\times10^5\ \text{N/m}^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

### Strategy

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find $p1$ .

### Solution

Solving Bernoulli's principle for $p_1$ yields

$$p_1=p_2+\frac{1}{2}\rho v_2^2-\frac{1}{2}\rho v_1^2=p_2+\frac{1}{2}\rho(v_2^2-v_1^2).$$

Substituting known values,

$$\begin{aligned} p_1 &= 1.01 \times 10^5 \text{ N/m}^2 + \frac{1}{2}(10^3 \text{ kg/m}^3)[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned}$$

### Significance

This absolute pressure in the hose is greater than in the nozzle, as expected, since $v$ is greater in the nozzle. The pressure $p_2$ in the nozzle must be atmospheric, because the water emerges into the atmosphere without other changes in conditions.

## Applications of Bernoulli's Principle

Many devices and situations occur in which fluid flows at a constant height and thus can be analyzed with Bernoulli's principle.

### Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times as pumps to raise water to small heights, as is necessary for draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in **Figure 14.31**.

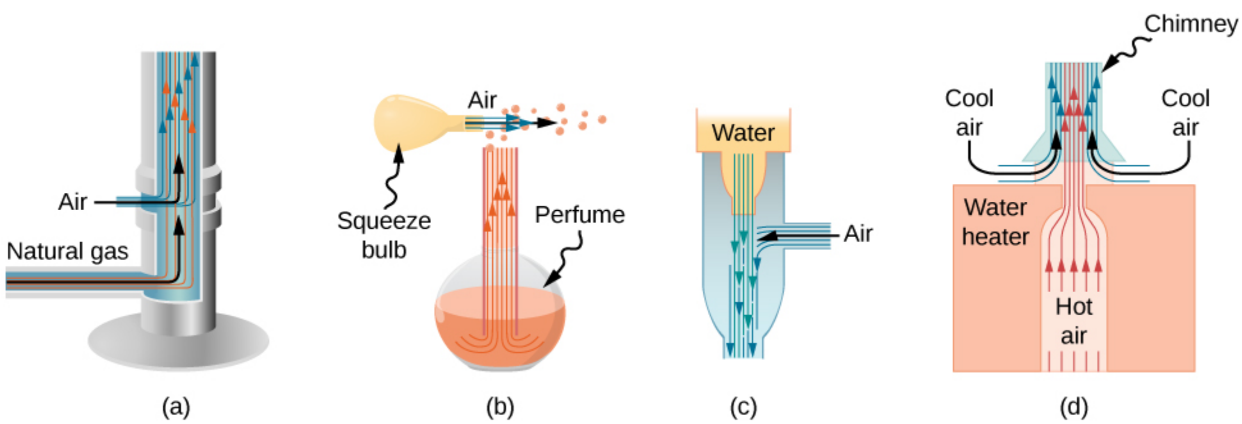



**Figure 14.31** Entrainment devices use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

### Velocity measurement

**Figure 14.32** shows two devices that apply Bernoulli's principle to measure fluid velocity. The manometer in part (a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ( $v_1 = 0$ ) in front of it, while fluid passing the other tube has velocity $v_2$ . This means that Bernoulli's principle as stated in

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

becomes

$$p_1 = p_2 + \frac{1}{2}\rho v_2^2.$$

Thus pressure $p_2$ over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, so the fluid in the manometer rises by $h$ on the side connected to the second opening, where

$$h \propto \frac{1}{2}\rho v_2^2.$$

(Recall that the symbol $\propto$ means "proportional to.") Solving for $v_2$, we see that

$$v_2 \propto \sqrt{h}.$$

Part (b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air-speed indicators in aircraft.

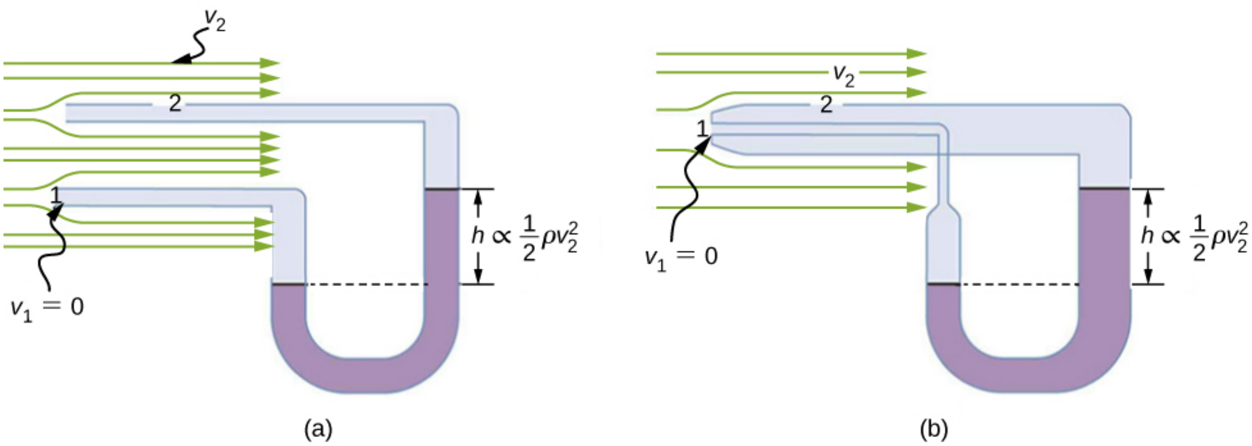



**Figure 14.32** Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, so the fluid has a speed *v* across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, so *h* is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

## A fire hose

All preceding applications of Bernoulli's equation involved simplifying conditions, such as constant height or constant pressure. The next example is a more general application of Bernoulli's equation in which pressure, velocity, and height all change.

## Example 14.7

### Calculating Pressure: A Fire Hose Nozzle

Fire hoses used in major structural fires have an inside diameter of 6.40 cm (**Figure 14.33**). Suppose such a hose carries a flow of 40.0 L/s, starting at a gauge pressure of $1.62 \times 10^6\ \text{N/m}^2$. The hose rises up 10.0 m along a ladder to a nozzle having an inside diameter of 3.00 cm. What is the pressure in the nozzle?

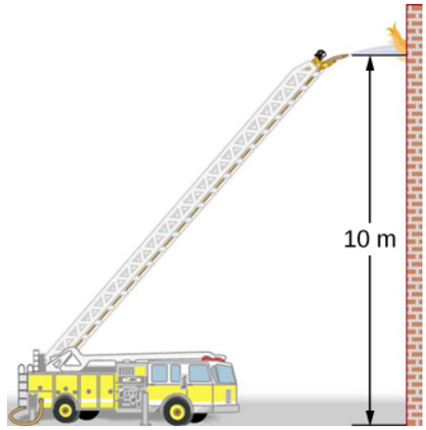



**Figure 14.33** Pressure in the nozzle of this fire hose is less than at ground level for two reasons: The water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its lowered pressure, the water can exert a large force on anything it strikes by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

### Strategy

We must use Bernoulli's equation to solve for the pressure, since depth is not constant.

### Solution

Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

where subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds $v_1$ and $v_2$. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3}\,\text{m}^3/\text{s}}{\pi(3.20 \times 10^{-2}\,\text{m})^2} = 12.4\text{m/s}.$$

Similarly, we find

$$v_2 = 56.6 \text{ m/s}.$$

This rather large speed is helpful in reaching the fire. Now, taking $h_1$ to be zero, we solve Bernoulli's equation for $p_2$:

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho g h_2.$$

Substituting known values yields

$$\begin{aligned} p_2 &= 1.62 \times 10^6 \text{ N/m}^2 + \frac{1}{2}(1000\,\text{kg/m}^3)[(12.4 \text{ m/s})^2 - (56.6 \text{ m/s})^2] \\ &- (1000\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(10.0\,\text{m}) \\ &= 0. \end{aligned}$$

### Significance

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus, the nozzle pressure equals atmospheric pressure as it must, because the water exits into the atmosphere without changes in its conditions.

# 14.7 | Viscosity and Turbulence

## Learning Objectives

By the end of this section, you will be able to:

- Explain what viscosity is
- Calculate flow and resistance with Poiseuille's law
- Explain how pressure drops due to resistance
- Calculate the Reynolds number for an object moving through a fluid
- Use the Reynolds number for a system to determine whether it is laminar or turbulent
- Describe the conditions under which an object has a terminal speed

In **Applications of Newton's Laws**, which introduced the concept of friction, we saw that an object sliding across the floor with an initial velocity and no applied force comes to rest due to the force of friction. Friction depends on the types of materials in contact and is proportional to the normal force. We also discussed drag and air resistance in that same chapter. We explained that at low speeds, the drag is proportional to the velocity, whereas at high speeds, drag is proportional to the velocity squared. In this section, we introduce the forces of friction that act on fluids in motion. For example, a fluid flowing through a pipe is subject to resistance, a type of friction, between the fluid and the walls. Friction also occurs between the different layers of fluid. These resistive forces affect the way the fluid flows through the pipe.

## Viscosity and Laminar Flow

When you pour yourself a glass of juice, the liquid flows freely and quickly. But if you pour maple syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity.

The precise definition of viscosity is based on laminar, or nonturbulent, flow. **Figure 14.34** shows schematically how laminar and turbulent flow differ. When flow is laminar, layers flow without mixing. When flow is turbulent, the layers mix, and significant velocities occur in directions other than the overall direction of flow.

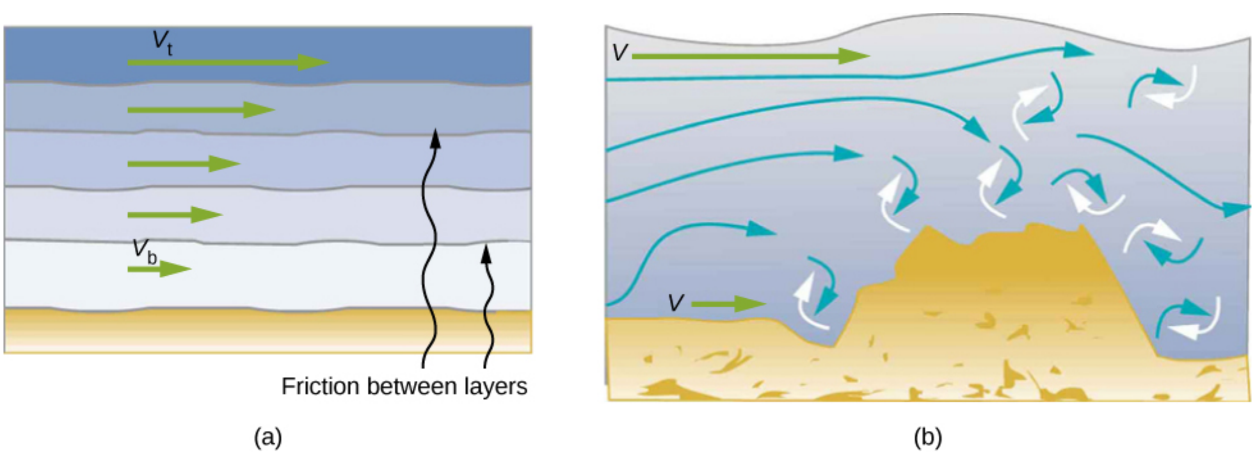



**Figure 14.34** (a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. The speed near the bottom of the flow ( $v_b$ ) is less than speed near the top ( $v_t$ ) because in this case, the surface of the containing vessel is at the bottom. (b) An obstruction in the vessel causes turbulent flow. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

**Turbulence** is a fluid flow in which layers mix together via eddies and swirls. It has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag between adjacent layers of fluid and between the fluid and its surroundings can form swirls and eddies if the speed is great enough. In **Figure 14.35**, the speed of the accelerating smoke reaches the point that it begins to swirl due to the drag between the smoke and the surrounding air.

**Figure 14.35** Smoke rises smoothly for a while and then begins to form swirls and eddies. The smooth flow is called laminar flow, whereas the swirls and eddies typify turbulent flow. Smoke rises more rapidly when flowing smoothly than after it becomes turbulent, suggesting that turbulence poses more resistance to flow. (credit: “Creativity103”/Flickr)

**Figure 14.36** shows how viscosity is measured for a fluid. The fluid to be measured is placed between two parallel plates. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, so the top layer moves at speed *v* while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from *v* to 0 as shown. Care is taken to ensure that the flow is laminar, that is, the layers do not mix. The motion in the figure is like a continuous shearing motion. Fluids have zero shear strength, but the rate at which they are sheared is related to the same geometrical factors *A* and *L* as is shear deformation for solids. In the diagram, the fluid is initially at rest. The layer of fluid in contact with the moving plate is accelerated and starts to move due to the internal friction between moving plate and the fluid. The next layer is in contact with the moving layer; since there is internal friction between the two layers, it also accelerates, and so on through the depth of the fluid. There is also internal friction between the stationary plate and the lowest layer of fluid, next to the station plate. The force is required to keep the plate moving at a constant velocity due to the internal friction.

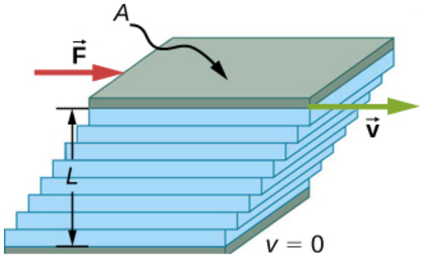



**Figure 14.36** Measurement of viscosity for laminar flow of fluid between two plates of area *A*. The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it.

A force *F* is required to keep the top plate in **Figure 14.36** moving at a constant velocity *v*, and experiments have shown that this force depends on four factors. First, *F* is directly proportional to *v* (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on *v*). Second, *F* is proportional to the area *A* of the plate. This relationship seems reasonable, since *A* is directly proportional to the amount of fluid being moved. Third, *F* is inversely proportional to the distance between the plates *L*. This relationship is also reasonable; *L* is like a lever arm, and the greater the lever arm, the less the force that is needed. Fourth, *F* is directly proportional to the coefficient of viscosity, $\eta$. The greater the viscosity, the greater the force required. These dependencies are combined into the equation

$$F = \eta \frac{vA}{L}.$$

This equation gives us a working definition of fluid viscosity $\eta$. Solving for $\eta$ gives

$$\eta = \frac{FL}{vA} \tag{14.17}$$

which defines viscosity in terms of how it is measured.

The SI unit of viscosity is $\text{N} \cdot \text{m}/\left[(\text{m/s})\text{m}^2\right] = \left(\text{N/m}^2\right)\text{s}$ or $\text{Pa} \cdot \text{s}$. **Table 14.4** lists the coefficients of viscosity for various fluids. Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities often depend on temperature.

| Fluid | Temperature (°C) | Viscosity $\eta$ (Pa · s) |
|---|---|---|
| Air | 0 | 0.0171 |
| | 20 | 0.0181 |
| | 40 | 0.0190 |
| | 100 | 0.0218 |
| Ammonia | 20 | 0.00974 |
| Carbon dioxide | 20 | 0.0147 |
| Helium | 20 | 0.0196 |
| Hydrogen | 0 | 0.0090 |
| Mercury | 20 | 0.0450 |
| Oxygen | 20 | 0.0203 |
| Steam | 100 | 0.0130 |
| Liquid water | 0 | 1.792 |
| | 20 | 1.002 |
| | 37 | 0.6947 |
| | 40 | 0.653 |
| | 100 | 0.282 |
| Whole blood | 20 | 3.015 |
| | 37 | 2.084 |
| Blood plasma | 20 | 1.810 |
| | 37 | 1.257 |
| Ethyl alcohol | 20 | 1.20 |
| Methanol | 20 | 0.584 |

**Table 14.4 Coefficients of Viscosity of Various Fluids**

| Fluid | Temperature (°C) | Viscosity $\eta$ (Pa · s) |
|---|---|---|
| Oil (heavy machine) | 20 | 660 |
| Oil (motor, SAE 10) | 30 | 200 |
| Oil (olive) | 20 | 138 |
| Glycerin | 20 | 1500 |
| Honey | 20 | 2000–10000 |
| Maple syrup | 20 | 2000–3000 |
| Milk | 20 | 3.0 |
| Oil (corn) | 20 | 65 |

**Table 14.4 Coefficients of Viscosity of Various Fluids**

## Laminar Flow Confined to Tubes: Poiseuille's Law

What causes flow? The answer, not surprisingly, is a pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate $Q$ is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as

$$Q = \frac{p_2 - p_1}{R}$$

where $p_1$ and $p_2$ are the pressures at two points, such as at either end of a tube, and $R$ is the resistance to flow. The resistance $R$ includes everything, except pressure, that affects flow rate. For example, $R$ is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of $R$. Turbulence greatly increases $R$, whereas increasing the diameter of a tube decreases $R$.

If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in a tube to viscous flow, as in **Figure 14.37**, we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame [part (c)], even though the viscosity of natural gas is small.

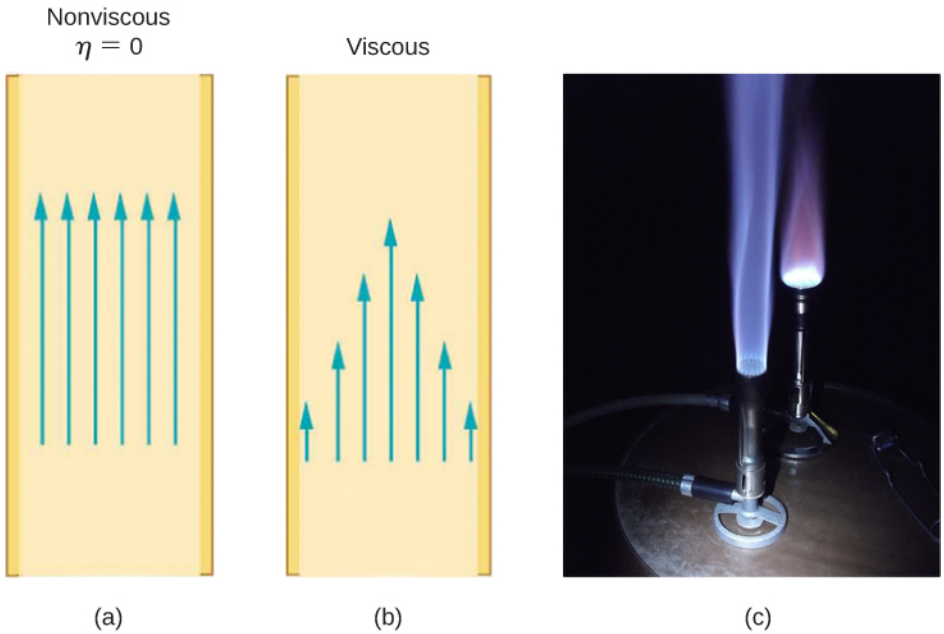



**Figure 14.37** (a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of a Bunsen burner flame is due to the velocity profile across the tube. (credit c: modification of work by Jason Woodhead)

The resistance $R$ to laminar flow of an incompressible fluid with viscosity $\eta$ through a horizontal tube of uniform radius $r$ and length $l$, is given by

$$R = \frac{8\eta l}{\pi r^4}. \tag{14.18}$$

This equation is called **Poiseuille's law for resistance**, named after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood through the body.

Let us examine Poiseuille's expression for $R$ to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity $\eta$ and the length $l$ of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius $r$ of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that $r$ is raised to the fourth power in Poiseuille's law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of $2^4 = 16$.

Taken together, $Q = \frac{p_2 - p_1}{R}$ and $R = \frac{8\eta l}{\pi r^4}$ give the following expression for flow rate:

$$Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}. \tag{14.19}$$

This equation describes laminar flow through a tube. It is sometimes called Poiseuille's law for laminar flow, or simply **Poiseuille's law** (Figure 14.38).

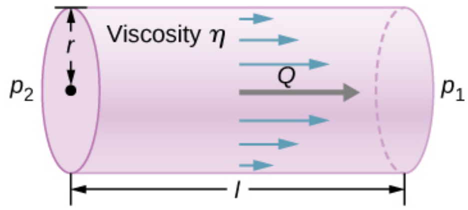



**Figure 14.38** Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity $\eta$ through a tube of length $l$ and radius $r$. The direction of flow is from greater to lower pressure. Flow rate $Q$ is directly proportional to the pressure difference $p_2 - p_1$, and inversely proportional to the length $l$ of the tube and viscosity $\eta$ of the fluid. Flow rate increases with radius by a factor of $r^4$.

## Example 14.8

### Using Flow Rate: Air Conditioning Systems

An air conditioning system is being designed to supply air at a gauge pressure of 0.054 Pa at a temperature of $20\,°\text{C}$. The air is sent through an insulated, round conduit with a diameter of 18.00 cm. The conduit is 20-meters long and is open to a room at atmospheric pressure 101.30 kPa. The room has a length of 12 meters, a width of 6 meters, and a height of 3 meters. (a) What is the volume flow rate through the pipe, assuming laminar flow? (b) Estimate the length of time to completely replace the air in the room. (c) The builders decide to save money by using a conduit with a diameter of 9.00 cm. What is the new flow rate?

### Strategy

Assuming laminar flow, Poiseuille's law states that

$$Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l} = \frac{dV}{dt}.$$

We need to compare the artery radius before and after the flow rate reduction. Note that we are given the diameter of the conduit, so we must divide by two to get the radius.

### Solution

a. Assuming a constant pressure difference and using the viscosity $\eta = 0.0181\ \text{mPa}\cdot\text{s}$,

$$Q = \frac{(0.054\ \text{Pa})(3.14)(0.09\ \text{m})^4}{8\left(0.0181 \times 10^{-3}\ \text{Pa}\cdot\text{s}\right)(20\ \text{m})} = 3.84 \times 10^{-3}\ \frac{\text{m}^3}{\text{s}}.$$

b. Assuming constant flow $Q = \frac{dV}{dt} \approx \frac{\Delta V}{\Delta t}$

$$\Delta t = \frac{\Delta V}{Q} = \frac{(12\ \text{m})(6\ \text{m})(3\ \text{m})}{3.84 \times 10^{-3}\ \frac{\text{m}^3}{\text{s}}} = 5.63 \times 10^4\ \text{s} = 15.63\ \text{hr}.$$

c. Using laminar flow, Poiseuille's law yields

$$Q = \frac{(0.054\ \text{Pa})(3.14)(0.045\ \text{m})^4}{8\left(0.0181 \times 10^{-3}\ \text{Pa}\cdot\text{s}\right)(20\ \text{m})} = 2.40 \times 10^{-4}\ \frac{\text{m}^3}{\text{s}}.$$

Thus, the radius of the conduit decreases by half reduces the flow rate to 6.25% of the original value.

### Significance

In general, assuming laminar flow, decreasing the radius has a more dramatic effect than changing the length. If the length is increased and all other variables remain constant, the flow rate is decreased:

$$\frac{Q_A}{Q_B} = \frac{\frac{(p_2 - p_1)\pi r_A^4}{8\eta l_A}}{\frac{(p_2 - p_1)\pi r_B^4}{8\eta l_B}} = \frac{l_B}{l_A}$$

$$Q_B = \frac{l_A}{l_B} Q_A.$$

Doubling the length cuts the flow rate to one-half the original flow rate.

If the radius is decreased and all other variables remain constant, the volume flow rate decreases by a much larger factor.

$$\frac{Q_A}{Q_B} = \frac{\frac{(p_2 - p_1)\pi r_A^4}{8\eta l_A}}{\frac{(p_2 - p_1)\pi r_B^4}{8\eta l_B}} = \left(\frac{r_A}{r_B}\right)^4$$

$$Q_B = \left(\frac{r_B}{r_A}\right)^4 Q_A$$

Cutting the radius in half decreases the flow rate to one-sixteenth the original flow rate.

## Flow and Resistance as Causes of Pressure Drops

Water pressure in homes is sometimes lower than normal during times of heavy use, such as hot summer days. The drop in pressure occurs in the water main before it reaches individual homes. Let us consider flow through the water main as illustrated in **Figure 14.39**. We can understand why the pressure $p_1$ to the home drops during times of heavy use by rearranging the equation for flow rate:

$$Q = \frac{p_2 - p_1}{R}$$

$$p_2 - p_1 = RQ.$$

In this case, $p_2$ is the pressure at the water works and $R$ is the resistance of the water main. During times of heavy use, the flow rate $Q$ is large. This means that $p_2 - p_1$ must also be large. Thus $p_1$ must decrease. It is correct to think of flow and resistance as causing the pressure to drop from $p_2$ to $p_1$. The equation $p_2 - p_1 = RQ$ is valid for both laminar and turbulent flows.

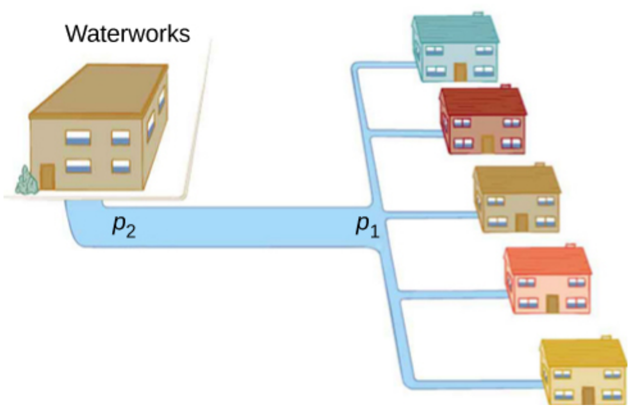



**Figure 14.39** During times of heavy use, there is a significant pressure drop in a water main, and $p_1$ supplied to users is significantly less than $p_2$ created at the water works. If the flow is very small, then the pressure drop is negligible, and $p_2 \approx p_1$.

We can also use $p_2 - p_1 = RQ$ to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance is much greater in narrow places, such as in an obstructed coronary artery. For a given flow rate $Q$, the pressure drop is greatest where the tube is most narrow. This is how water faucets control flow. Additionally, $R$ is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

## Measuring Turbulence

An indicator called the **Reynolds number** $N_R$ can reveal whether flow is laminar or turbulent. For flow in a tube of uniform diameter, the Reynolds number is defined as

$$N_R = \frac{2\rho vr}{\eta} \text{(fl w in tube)} \quad \textbf{(14.20)}$$

where $\rho$ is the fluid density, $v$ its speed, $\eta$ its viscosity, and $r$ the tube radius. The Reynolds number is a dimensionless quantity. Experiments have revealed that $N_R$ is related to the onset of turbulence. For $N_R$ below about 2000, flow is laminar. For $N_R$ above about 3000, flow is turbulent.

For values of $N_R$ between about 2000 and 3000, flow is unstable—that is, it can be laminar, but small obstructions and surface roughness can make it turbulent, and it may oscillate randomly between being laminar and turbulent. In fact, the flow of a fluid with a Reynolds number between 2000 and 3000 is a good example of chaotic behavior. A system is defined to be chaotic when its behavior is so sensitive to some factor that it is extremely difficult to predict. It is difficult, but not impossible, to predict whether flow is turbulent or not when a fluid's Reynold's number falls in this range due to extremely sensitive dependence on factors like roughness and obstructions on the nature of the flow. A tiny variation in one factor has an exaggerated (or nonlinear) effect on the flow.

### Example 14.9

#### Using Flow Rate: Turbulent Flow or Laminar Flow

In **Example 14.8**, we found the volume flow rate of an air conditioning system to be $Q = 3.84 \times 10^{-3}\ \text{m}^3/\text{s}$. This calculation assumed laminar flow. (a) Was this a good assumption? (b) At what velocity would the flow become turbulent?

#### Strategy

To determine if the flow of air through the air conditioning system is laminar, we first need to find the velocity, which can be found by

$$Q = Av = \pi r^2 v.$$

Then we can calculate the Reynold's number, using the equation below, and determine if it falls in the range for laminar flow

$$R = \frac{2\rho vr}{\eta}.$$

#### Solution

a. Using the values given:

$$v = \frac{Q}{\pi r^2} = \frac{3.84 \times 10^{-3}\ \frac{\text{m}^3}{\text{s}}}{3.14(0.09\ \text{m})^2} = 0.15\frac{\text{m}}{\text{s}}$$

$$R = \frac{2\rho vr}{\eta} = \frac{2\left(1.23\frac{\text{kg}}{\text{m}^3}\right)(0.15\ \frac{\text{m}}{\text{s}})(0.09\ \text{m})}{0.0181 \times 10^{-3}\ \text{Pa} \cdot \text{s}} = 1835.$$

Since the Reynolds number is 1835 < 2000, the flow is laminar and not turbulent. The assumption that the flow was laminar is valid.

b. To find the maximum speed of the air to keep the flow laminar, consider the Reynold's number.

$$\begin{aligned} R &= \frac{2\rho v r}{\eta} \leq 2000 \\ v &= \frac{2000\left(0.0181 \times 10^{-3}\,\text{Pa} \cdot \text{s}\right)}{2\left(1.23\frac{\text{kg}}{\text{m}^3}\right)(0.09\text{ m})} = 0.16\frac{\text{m}}{\text{s}}. \end{aligned}$$

### Significance

When transferring a fluid from one point to another, it desirable to limit turbulence. Turbulence results in wasted energy, as some of the energy intended to move the fluid is dissipated when eddies are formed. In this case, the air conditioning system will become less efficient once the velocity exceeds 0.16 m/s, since this is the point at which turbulence will begin to occur.

# CHAPTER 14 REVIEW

## KEY TERMS

**absolute pressure** sum of gauge pressure and atmospheric pressure

**Archimedes' principle** buoyant force on an object equals the weight of the fluid it displaces

**Bernoulli's equation** equation resulting from applying conservation of energy to an incompressible frictionless fluid:
$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}, \quad \text{throughout the fluid}$

**Bernoulli's principle** Bernoulli's equation applied at constant depth:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

**buoyant force** net upward force on any object in any fluid due to the pressure difference at different depths

**density** mass per unit volume of a substance or object

**flow rate** abbreviated $Q$, it is the volume $V$ that flows past a particular point during a time $t$, or $Q = dV/dt$

**fluids** liquids and gases; a fluid is a state of matter that yields to shearing forces

**gauge pressure** pressure relative to atmospheric pressure

**hydraulic jack** simple machine that uses cylinders of different diameters to distribute force

**hydrostatic equilibrium** state at which water is not flowing, or is static

**ideal fluid** fluid with negligible viscosity

**laminar flow** type of fluid flow in which layers do not mix

**Pascal's principle** change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

**Poiseuille's law** rate of laminar flow of an incompressible fluid in a tube: $Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}$.

**Poiseuille's law for resistance** resistance to laminar flow of an incompressible fluid in a tube: $R = \frac{8\eta l}{\pi r^4}$

**pressure** force per unit area exerted perpendicular to the area over which the force acts

**Reynolds number** dimensionless parameter that can reveal whether a particular flow is laminar or turbulent

**specific gravity** ratio of the density of an object to a fluid (usually water)

**turbulence** fluid flow in which layers mix together via eddies and swirls

**turbulent flow** type of fluid flow in which layers mix together via eddies and swirls

**viscosity** measure of the internal friction in a fluid

## KEY EQUATIONS

| | |
|---|---|
| Density of a sample at constant density | $\rho = \frac{m}{V}$ |
| Pressure | $p = \frac{F}{A}$ |
| Pressure at a depth *h* in a fluid of constant density | $p = p_0 + \rho gh$ |
| Change of pressure with height in a constant-density fluid | $\frac{dp}{dy} = -\rho g$ |

| | |
|---|---|
| Absolute pressure | $p_{\text{abs}} = p_{\text{g}} + p_{\text{atm}}$ |
| Pascal's principle | $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ |
| Volume flow rate | $Q = \frac{dV}{dt}$ |
| Continuity equation (constant density) | $A_1 v_1 = A_2 v_2$ |
| Continuity equation (general form) | $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ |
| Bernoulli's equation | $p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$ |
| Viscosity | $\eta = \frac{FL}{vA}$ |
| Poiseuille's law for resistance | $R = \frac{8\eta l}{\pi r^4}$ |
| Poiseuille's law | $Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}$ |

# SUMMARY

## 14.1 Fluids, Density, and Pressure

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.
- Density is the mass per unit volume of a substance or object, defined as $\rho = m/V$. The SI unit of density is $\text{kg/m}^3$.
- Pressure is the force per unit perpendicular area over which the force is applied, $p = F/A$. The SI unit of pressure is the pascal: $1\ \text{Pa} = 1\ \text{N/m}^2$.
- Pressure due to the weight of a liquid of constant density is given by $p = \rho gh$, where $p$ is the pressure, $h$ is the depth of the liquid, $\rho$ is the density of the liquid, and $g$ is the acceleration due to gravity.

## 14.2 Measuring Pressure

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Open-tube manometers have U-shaped tubes and one end is always open. They are used to measure pressure. A mercury barometer is a device that measures atmospheric pressure.
- The SI unit of pressure is the pascal (Pa), but several other units are commonly used.

## 14.3 Pascal's Principle and Hydraulics

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

## 14.4 Archimedes' Principle and Buoyancy

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object can remain suspended at its present depth. The buoyant force is always present and acting on any object immersed either partially or entirely in a fluid.

- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.

### 14.5 Fluid Dynamics

- Flow rate $Q$ is defined as the volume $V$ flowing past a point in time $t$, or $Q = \frac{dV}{dt}$ where $V$ is volume and $t$ is time. The SI unit of flow rate is $\text{m}^3/\text{s}$, but other rates can be used, such as L/min.
- Flow rate and velocity are related by $Q = Av$ where $A$ is the cross-sectional area of the flow and $v$ is its average velocity.
- The equation of continuity states that for an incompressible fluid, the mass flowing into a pipe must equal the mass flowing out of the pipe.

### 14.6 Bernoulli's Equation

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which the height of the fluid is constant. The terms involving depth (or height $h$) subtract out, yielding

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2.$$

- Bernoulli's principle has many applications, including entrainment and velocity measurement.

### 14.7 Viscosity and Turbulence

- Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.
- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity $\eta$ is due to friction within a fluid.
- Flow is proportional to pressure difference and inversely proportional to resistance:

$$Q = \frac{p - 2p_1}{R}.$$

- The pressure drop caused by flow and resistance is given by $p_2 - p_1 = RQ$.
- The Reynolds number $N_R$ can reveal whether flow is laminar or turbulent. It is $N_R = \frac{2\rho v r}{\eta}$.
- For $N_R$ below about 2000, flow is laminar. For $N_R$ above about 3000, flow is turbulent. For values of $N_R$ between 2000 and 3000, it may be either or both.

## CONCEPTUAL QUESTIONS

### 14.1 Fluids, Density, and Pressure

**1.** Which of the following substances are fluids at room temperature and atmospheric pressure: air, mercury, water, glass?

**2.** Why are gases easier to compress than liquids and solids?

**3.** Explain how the density of air varies with altitude.

**4.** The image shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

**5.** How is pressure related to the sharpness of a knife and its ability to cut?

**6.** Why is a force exerted by a static fluid on a surface always perpendicular to the surface?

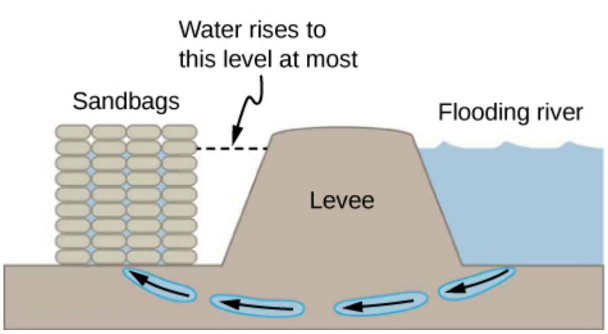
**7.** Imagine that in a remote location near the North Pole, a chunk of ice floats in a lake. Next to the lake, a glacier with the same volume as the floating ice sits on land. If both chunks of ice should melt due to rising global temperatures, and the melted ice all goes into the lake, which one would cause the level of the lake to rise the most? Explain.

**8.** In ballet, dancing *en pointe* (on the tips of the toes) is much harder on the toes than normal dancing or walking. Explain why, in terms of pressure.

**9.** Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

**10.** Why does atmospheric pressure decrease more rapidly than linearly with altitude?

**11.** The image shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column of sandbags is able to balance the much larger body of water behind the levee.




**12.** Is there a net force on a dam due to atmospheric pressure? Explain your answer.

**13.** Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure not affect the total pressure in a fluid?

**14.** You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks only if there is no air between the cork and liquid.

## 14.2 Measuring Pressure

**15.** Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.

## 14.3 Pascal's Principle and Hydraulics

**16.** Suppose the master cylinder in a hydraulic system is at a greater height than the cylinder it is controlling. Explain how this will affect the force produced at the cylinder that is being controlled.

## 14.4 Archimedes' Principle and Buoyancy

**17.** More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

**18.** Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

**19.** Will the same ship float higher in salt water than in freshwater? Explain your answer.

**20.** Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

## 14.5 Fluid Dynamics

**21.** Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (*Hint:* Consider the relationship between fluid velocity and the cross-sectional area through which the fluid flows.)

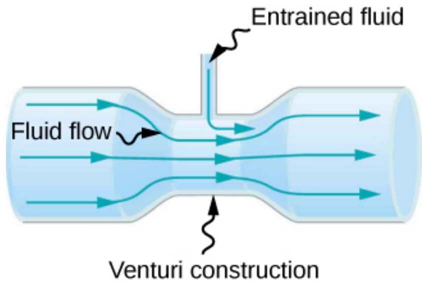
## 14.6 Bernoulli's Equation

**22.** You can squirt water from a garden hose a considerably greater distance by partially covering the opening with your thumb. Explain how this works.

**23.** Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why.

**24.** Look back to **Figure 14.29**. Answer the following two questions. Why is $p_o$ less than atmospheric? Why is $p_o$ greater than $p_i$?

**25.** A tube with a narrow segment designed to enhance entrainment is called a Venturi, such as shown below. Venturis are very commonly used in carburetors and aspirators. How does this structure bolster entrainment?




**26.** Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.

**27.** Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

**28.** Why is it preferable for airplanes to take off into the wind rather than with the wind?

**29.** Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.

**30.** It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

**31.** Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

**32.** David rolled down the window on his car while driving on the freeway. An empty plastic bag on the floor promptly flew out the window. Explain why.

**33.** Based on Bernoulli's equation, what are three forms of energy in a fluid? (Note that these forms are conservative, unlike heat transfer and other dissipative forms not included in Bernoulli's equation.)

**34.** The old rubber boot shown below has two leaks. To what maximum height can the water squirt from Leak 1? How does the velocity of water emerging from Leak 2 differ from that of Leak 1? Explain your responses in terms of energy.


**35.** Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

## 14.7 Viscosity and Turbulence

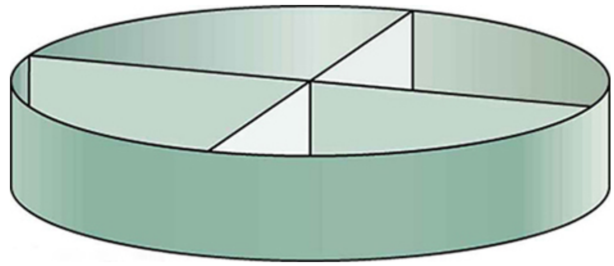
**36.** Explain why the viscosity of a liquid decreases with temperature, that is, how might an increase in temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature, that is, how does increased gas temperature create more collisions between atoms and molecules?

**37.** When paddling a canoe upstream, it is wisest to travel as near to the shore as possible. When canoeing downstream, it is generally better to stay near the middle. Explain why.

**38.** Plumbing usually includes air-filled tubes near water faucets (see the following figure). Explain why they are needed and how they work.

**39.** Doppler ultrasound can be used to measure the speed of blood in the body. If there is a partial constriction of an artery, where would you expect blood speed to be greatest: at or after the constriction? What are the two distinct causes of higher resistance in the constriction?

**40.** Sink drains often have a device such as that shown below to help speed the flow of water. How does this work?



# PROBLEMS

## 14.1 Fluids, Density, and Pressure

**41.** Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

**42.** Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb.). What is the volume in liters of this much mercury?

**43.** What is the mass of a deep breath of air having a volume of 2.00 L? Discuss the effect taking such a breath has on your body's volume and density.

**44.** A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces $89.0\,\text{cm}^3$ of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

**45.** Suppose you have a coffee mug with a circular cross-section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

**46.** A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

**47.** A trash compactor can compress its contents to 0.350 times their original volume. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

**48.** A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?

**49.** What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

**50.** The tip of a nail exerts tremendous pressure when hit by a hammer because it exerts a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00-mm diameter to create a pressure of $3.00 \times 10^9\ \text{N/m}^2$? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

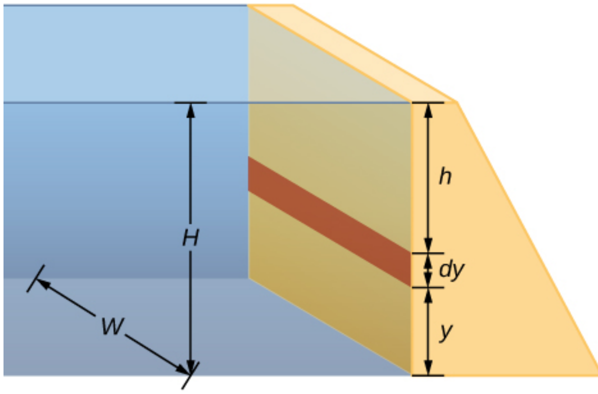
**51.** A glass tube contains mercury. What would be the height of the column of mercury which would create pressure equal to 1.00 atm?

**52.** The greatest ocean depths on Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

**53.** Verify that the SI unit of $h\rho g$ is $\text{N/m}^2$.

**54.** What pressure is exerted on the bottom of a gas tank that is 0.500-m wide and 0.900-m long and can hold 50.0 kg of gasoline when full?

**55.** A dam is used to hold back a river. The dam has a height $H = 12\,\text{m}$ and a width $W = 10\,\text{m}$. Assume that the density of the water is $\rho = 1000\,\text{kg/m}^3$. (a) Determine the net force on the dam. (b) Why does the thickness of the dam increase with depth?




## 14.2 Measuring Pressure

**56.** Find the gauge and absolute pressures in the balloon and peanut jar shown in **Figure 14.12**, assuming the manometer connected to the balloon uses water and the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking $h = 0.0500\text{m}$ for each.

**57.** How tall must a water-filled manometer be to measure blood pressure as high as 300 mm Hg?

**58.** Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground if a bicycle and rider have a total mass of 80.0 kg, and the gauge pressure in the tires is $3.50 \times 10^5$ Pa .

## 14.3 Pascal's Principle and Hydraulics

**59.** How much pressure is transmitted in the hydraulic system considered in **Example 14.3**? Express your answer in atmospheres.

**60.** What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on a second cylinder? The master cylinder has a 2.00-cm diameter and the second cylinder has a 24.0-cm diameter.

**61.** A host pours the remnants of several bottles of wine into a jug after a party. The host then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. The host is amazed when the host pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

**62.** A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the cylinder that is being controlled to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses due to friction.

**63.** Verify that work input equals work output for a hydraulic system assuming no losses due to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

## 14.4 Archimedes' Principle and Buoyancy

**64.** What fraction of ice is submerged when it floats in freshwater, given the density of water at $0\,°\text{C}$ is very close to $1000\,\text{kg/m}^3$ ?

**65.** If a person's body has a density of $995\,\text{kg/m}^3$ , what fraction of the body will be submerged when floating gently in (a) freshwater? (b) In salt water with a density of $1027\,\text{kg/m}^3$ ?

**66.** A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

**67.** Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in **Table 14.1**? (c) Calculate the fluid's density and identify it.

**68.** Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? Neglect the volume of the rubber.

**69.** What is the density of a woman who floats in fresh water with $4.00\%$ of her volume above the surface? (This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water.) (b) What percent of her volume is above the surface when she floats in seawater?

**70.** A man has a mass of 80 kg and a density of $955 \text{kg/m}^3$ (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

**71.** A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

**72.** What percentage of an iron anchor's weight will be supported by buoyant force when submerged in salt water?

**73.** Referring to **Figure 14.20**, prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas $A$. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

**74.** A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

## 14.5 Fluid Dynamics

**75.** What is the average flow rate in $\text{cm}^3/\text{s}$ of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

**76.** The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to $\text{cm}^3/\text{s}$. (b) What is this rate in $\text{m}^3/\text{s}$?

**77.** The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions. On average, the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20-m wide and averages 20-m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

**78.** (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take if you could divert a moderate size river, flowing at $5000 \text{ m}^3/\text{s}$ into the pool?

**79.** What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

**80.** Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

**81.** Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

**82.** Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in $\text{cm}^3/\text{s}$? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

## 14.6 Bernoulli's Equation

**83.** Verify that pressure has units of energy per unit volume.

**84.** Suppose you have a wind speed gauge like the pitot tube shown in **Figure 14.32**. By what factor must wind speed increase to double the value of *h* in the manometer? Is this independent of the moving fluid and the fluid in the manometer?

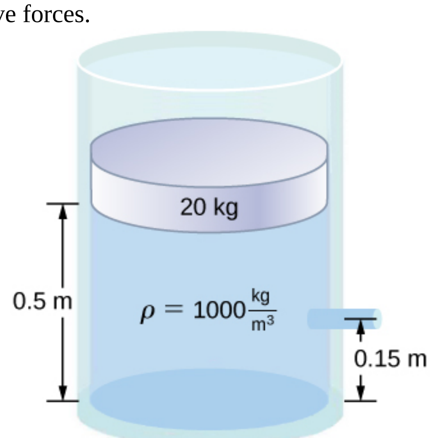
**85.** If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?

**86.** Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mph) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli equation on a roof having an area of $220 \text{m}^2$? Typical air density in Boulder is $1.14 \text{kg/m}^3$, and the corresponding atmospheric pressure is $8.89 \times 10^4 \text{N/m}^2$. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

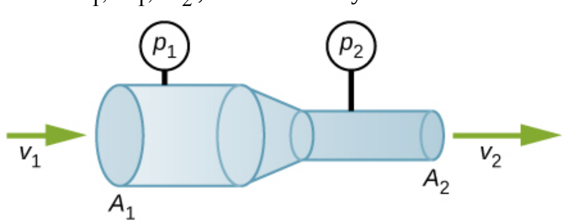
**87.** What is the pressure drop due to the Bernoulli Effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

**88.** (a) Using Bernoulli's equation, show that the measured fluid speed $v$ for a pitot tube, like the one in Figure 14.32(b), is given by $v = \left(\frac{2\rho' gh}{\rho}\right)^{1/2}$, where $h$ is the height of the manometer fluid, $\rho'$ is the density of the manometer fluid, $\rho$ is the density of the moving fluid, and $g$ is the acceleration due to gravity. (Note that $v$ is indeed proportional to the square root of $h$, as stated in the text.) (b) Calculate $v$ for moving air if a mercury manometer's $h$ is 0.200 m.

**89.** A container of water has a cross-sectional area of $A = 0.1\ \text{m}^2$. A piston sits on top of the water (see the following figure). There is a spout located 0.15 m from the bottom of the tank, open to the atmosphere, and a stream of water exits the spout. The cross sectional area of the spout is $A_s = 7.0 \times 10^{-4}\ \text{m}^2$. (a) What is the velocity of the water as it leaves the spout? (b) If the opening of the spout is located 1.5 m above the ground, how far from the spout does the water hit the floor? Ignore all friction and dissipative forces.




**90.** A fluid of a constant density flows through a reduction in a pipe. Find an equation for the change in pressure, in terms of $v_1$, $A_1$, $A_2$, and the density.




## 14.7 Viscosity and Turbulence

**91.** (a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information: air temperature is $20\ °\text{C}$, the cart is moving at 0.400 m/s, its surface area is $2.50 \times 10^{-2}\ \text{m}^2$, and the thickness of the air layer is $6.00 \times 10^{-5}\ \text{m}$. (b) What is the ratio of this force to the weight of the 0.300-kg cart?

**92.** The arterioles (small arteries) leading to an organ constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor do the radii of the arterioles constrict?

**93.** A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $F_s = 6\pi r\eta v$. Show that the terminal speed is given by $v = \frac{2R^2 g}{9\eta}(\rho_s - \rho_1)$, where $R$ is the radius of the sphere, $\rho_s$ is its density, and $\rho_1$ is the density of the fluid, and $\eta$ the coefficient of viscosity.

**94.** Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.

**95.** A skydiver will reach a terminal velocity when the air drag equals his or her weight. For a skydiver with a large body, turbulence is a factor at high speeds. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be $F_D = \frac{1}{2}\rho A v^2$, and setting this equal to the skydiver's weight, find the terminal speed for a person falling "spread eagle."

**96.** (a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?

**97.** When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?

**98.** An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be $900\,\text{kg/m}^3$ and its viscosity to be $1.00(\text{N/m}^2)\cdot\text{s}$ (or $1.00\,\text{Pa}\cdot\text{s}$). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.

**99.** Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6\ \text{N/m}^2$. (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete's velocity.

**100.** Verify that the flow of oil is laminar for an oil gusher that shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. The vertical pipe is 50 m long. Take the density of the oil to be $900\,\text{kg/m}^3$ and its viscosity to be $1.00(\text{N/m}^2)\cdot\text{s}$ (or $1.00\,\text{Pa}\cdot\text{s}$).

**101.** Calculate the Reynolds numbers for the flow of water through (a) a nozzle with a radius of 0.250 cm and (b) a garden hose with a radius of 0.900 cm, when the nozzle is attached to the hose. The flow rate through hose and nozzle is 0.500 L/s. Can the flow in either possibly be laminar?

**102.** A fire hose has an inside diameter of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6\ \text{N/m}^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Calculate the Reynolds numbers for flow in the fire hose and nozzle to show that the flow in each must be turbulent.

**103.** At what flow rate might turbulence begin to develop in a water main with a 0.200-m diameter? Assume a $20\,°\text{C}$ temperature.

## ADDITIONAL PROBLEMS

**104.** Before digital storage devices, such as the memory in your cell phone, music was stored on vinyl disks with grooves with varying depths cut into the disk. A phonograph used a needle, which moved over the grooves, measuring the depth of the grooves. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle with a 0.200-mm radius, what pressure is exerted on the record in Pa?

**105.** Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5\ \text{N/m}^2$?

**106.** The aqueous humor in a person's eye is exerting a force of 0.300 N on the $1.10\text{-cm}^2$ area of the cornea. What pressure is this in mm Hg?

**107.** (a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid $(p = h\rho g)$ rather than a conversion factor. (b) Explain why the blood pressure of an infant would likely be smaller than that of an adult. Specifically, consider the smaller height to which blood must be pumped.

**108.** Pressure cookers have been around for more than 300 years, although their use has greatly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is $25.0\,\text{cm}$ in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

**109.** Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (assume the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

**110.** In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

**111.** Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is $1015\,\text{kg/m}^3$?

**112.** The human circulation system has approximately $1 \times 10^9$ capillary vessels. Each vessel has a diameter of about $8 \mu m$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

**113.** The flow rate of blood through a $2.00 \times 10^{-6}$ m -radius capillary is $3.80 \times 10^9$ $cm^3/s$. (a) What is the speed of the blood flow? (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0 \; cm^3/s$ ?

**114.** The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0 \; cm^3/s$, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.

**115.** A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of $3.00 \times 10^5$ $N/m^2$. (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

**116.** A glucose solution being administered with an IV has a flow rate of $4.00 \; cm^3/min$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.

**117.** A small artery has a length of $1.1 \times 10^{-3}$ m and a radius of $2.5 \times 10^{-5}$ m. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is $37 \; °C$.)

**118.** Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?

**119.** Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

## CHALLENGE PROBLEMS

**120.** The pressure on the dam shown early in the problems section increases with depth. Therefore, there is a net torque on the dam. Find the net torque.

**121.** The temperature of the atmosphere is not always constant and can increase or decrease with height. In a neutral atmosphere, where there is not a significant amount of vertical mixing, the temperature decreases at a rate of approximately 6.5 K per km. The magnitude of the decrease in temperature as height increases is known as the lapse rate ($\Gamma$). (The symbol is the upper case Greek letter gamma.) Assume that the surface pressure is $p_0 = 1.013 \times 10^5$ Pa where $T = 293$ K and the lapse rate is $\left(-\Gamma = 6.5 \frac{K}{km}\right)$. Estimate the pressure 3.0 km above the surface of Earth.

**122.** A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.

**123.** Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

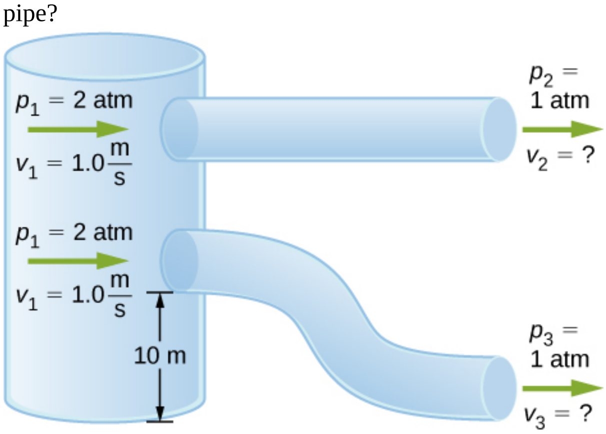
**124.** Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them at prices much below gold value but high above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

**125.** The inside volume of a house is equivalent to that of a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high. The house is heated by a forced air gas heater. The main uptake air duct of the heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 minutes?

**126.** A garden hose with a diameter of 2.0 cm is used to fill a bucket, which has a volume of 0.10 cubic meters. It takes 1.2 minutes to fill. An adjustable nozzle is attached to the hose to decrease the diameter of the opening, which increases the speed of the water. The hose is held level to the ground at a height of 1.0 meters and the diameter is decreased until a flower bed 3.0 meters away is reached. (a) What is the volume flow rate of the water through the nozzle when the diameter is 2.0 cm? (b) What is the speed of the water coming out of the hose? (c) What does the speed of the water coming out of the hose need to be to reach the flower bed 3.0 meters away? (d) What is the diameter of the nozzle needed to reach the flower bed?

**127.** A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air as $1.29\,\text{kg/m}^3$, how fast must it move over the upper surface to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth that at sea level? (Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)

**128.** Two pipes of equal and constant diameter leave a water pumping station and dump water out of an open end that is open to the atmosphere (see the following figure). The water enters at a pressure of two atmospheres and a speed of $(\text{v}_1 = 1.0\,\text{m/s})$. One pipe drops a height of 10 m. What is the velocity of the water as the water leaves each pipe?




**129.** Fluid originally flows through a tube at a rate of $100\,\text{cm}^3/\text{s}$. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, and the pressure difference is increased by a factor of 1.50.

**130.** During a marathon race, a runner's blood flow increases to 10.0 times her resting rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

**131.** Water supplied to a house by a water main has a pressure of $3.00 \times 10^5\ \text{N/m}^2$ early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is $5.00 \times 10^5\ \text{N/m}^2$, and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?

**132.** Gasoline is piped underground from refineries to major users. The flow rate is $3.00 \times 10^{-2}\ \text{m}^3/\text{s}$ (about 500 gal/min), the viscosity of gasoline is $1.00 \times 10^{-3}\ (\text{N/m}^2) \cdot \text{s}$, and its density is $680\,\text{kg/m}^3$. (a) What minimum diameter must the pipe have if the Reynolds number is to be less than 2000? (b) What pressure difference must be maintained along each kilometer of the pipe to maintain this flow rate?