

# 1 | TEMPERATURE AND HEAT



**Figure 1.1** These snowshoers on Mount Hood in Oregon are enjoying the heat flow and light caused by high temperature. All three mechanisms of heat transfer are relevant to this picture. The heat flowing out of the fire also turns the solid snow to liquid water and vapor. (credit: “Mt. Hood Territory”/Flickr)

## Chapter Outline

- 1.1 Temperature and Thermal Equilibrium
- 1.2 Thermometers and Temperature Scales
- 1.3 Thermal Expansion
- 1.4 Heat Transfer, Specific Heat, and Calorimetry
- 1.5 Phase Changes
- 1.6 Mechanisms of Heat Transfer

## Introduction

Heat and temperature are important concepts for each of us, every day. How we dress in the morning depends on whether the day is hot or cold, and most of what we do requires energy that ultimately comes from the Sun. The study of heat and temperature is part of an area of physics known as thermodynamics. The laws of thermodynamics govern the flow of energy throughout the universe. They are studied in all areas of science and engineering, from chemistry to biology to environmental science.

In this chapter, we explore heat and temperature. It is not always easy to distinguish these terms. Heat is the flow of energy from one object to another. This flow of energy is caused by a difference in temperature. The transfer of heat can change temperature, as can work, another kind of energy transfer that is central to thermodynamics. We return to these basic ideas several times throughout the next four chapters, and you will see that they affect everything from the behavior of atoms and molecules to cooking to our weather on Earth to the life cycles of stars.

## 1.1 | Temperature and Thermal Equilibrium

### Learning Objectives

By the end of this section, you will be able to:

- Define temperature and describe it qualitatively
- Explain thermal equilibrium
- Explain the zeroth law of thermodynamics

Heat is familiar to all of us. We can feel heat entering our bodies from the summer Sun or from hot coffee or tea after a winter stroll. We can also feel heat leaving our bodies as we feel the chill of night or the cooling effect of sweat after exercise.

What is heat? How do we define it and how is it related to temperature? What are the effects of heat and how does it flow from place to place? We will find that, in spite of the richness of the phenomena, a small set of underlying physical principles unites these subjects and ties them to other fields. We start by examining temperature and how to define and measure it.

### Temperature

The concept of temperature has evolved from the common concepts of hot and cold. The scientific definition of temperature explains more than our senses of hot and cold. As you may have already learned, many physical quantities are defined solely in terms of how they are observed or measured, that is, they are defined *operationally*. **Temperature** is operationally defined as the quantity of what we measure with a thermometer. As we will see in detail in a later chapter on the kinetic theory of gases, temperature is proportional to the average kinetic energy of translation, a fact that provides a more physical definition. Differences in temperature maintain the transfer of heat, or *heat transfer*, throughout the universe. **Heat transfer** is the movement of energy from one place or material to another as a result of a difference in temperature. (You will learn more about heat transfer later in this chapter.)

### Thermal Equilibrium

An important concept related to temperature is **thermal equilibrium**. Two objects are in thermal equilibrium if they are in close contact that allows either to gain energy from the other, but nevertheless, no net energy is transferred between them. Even when not in contact, they are in thermal equilibrium if, when they are placed in contact, no net energy is transferred between them. If two objects remain in contact for a long time, they typically come to equilibrium. In other words, two objects in thermal equilibrium do not exchange energy.

Experimentally, if object *A* is in equilibrium with object *B*, and object *B* is in equilibrium with object *C*, then (as you may have already guessed) object *A* is in equilibrium with object *C*. That statement of transitivity is called the **zeroth law of thermodynamics**. (The number “zeroth” was suggested by British physicist Ralph Fowler in the 1930s. The first, second, and third laws of thermodynamics were already named and numbered then. The zeroth law had seldom been stated, but it needs to be discussed before the others, so Fowler gave it a smaller number.) Consider the case where *A* is a thermometer. The zeroth law tells us that if *A* reads a certain temperature when in equilibrium with *B*, and it is then placed in contact with *C*, it will not exchange energy with *C*; therefore, its temperature reading will remain the same (**Figure 1.2**). In other words, *if two objects are in thermal equilibrium, they have the same temperature*.



**Figure 1.2** If thermometer *A* is in thermal equilibrium with object *B*, and *B* is in thermal equilibrium with *C*, then *A* is in thermal equilibrium with *C*. Therefore, the reading on *A* stays the same when *A* is moved over to make contact with *C*.

A thermometer measures its own temperature. It is through the concepts of thermal equilibrium and the zeroth law of thermodynamics that we can say that a thermometer measures the temperature of *something else*, and to make sense of the statement that two objects are at the same temperature.

In the rest of this chapter, we will often refer to “systems” instead of “objects.” As in the chapter on linear momentum and collisions, a system consists of one or more objects—but in thermodynamics, we require a system to be macroscopic, that is, to consist of a huge number (such as  $10^{23}$ ) of molecules. Then we can say that a system is in thermal equilibrium with itself if all parts of it are at the same temperature. (We will return to the definition of a thermodynamic system in the chapter on the first law of thermodynamics.)

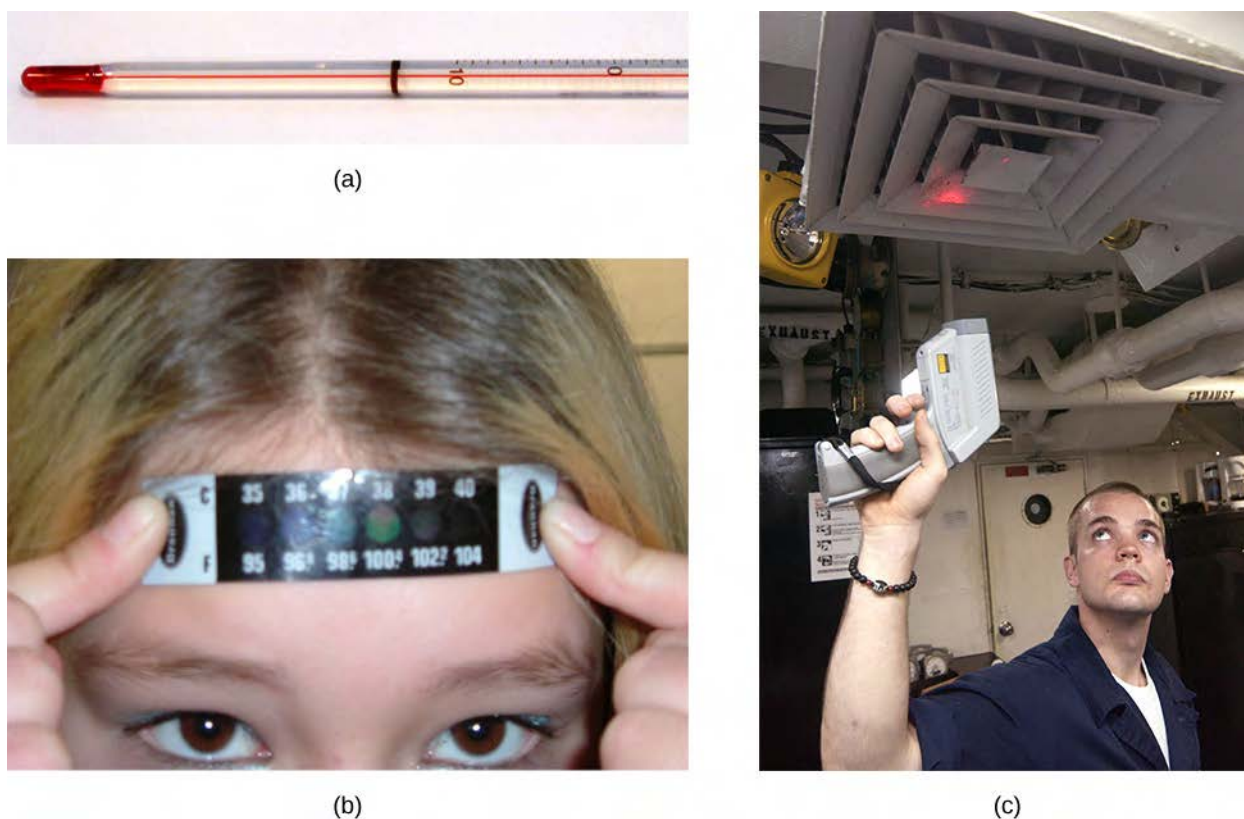
## 1.2 | Thermometers and Temperature Scales

### Learning Objectives

By the end of this section, you will be able to:

- Describe several different types of thermometers
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales

Any physical property that depends consistently and reproducibly on temperature can be used as the basis of a thermometer. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer and the original mercury thermometers. Other properties used to measure temperature include electrical resistance, color, and the emission of infrared radiation (**Figure 1.3**).



**Figure 1.3** Because many physical properties depend on temperature, the variety of thermometers is remarkable. (a) In this common type of thermometer, the alcohol, containing a red dye, expands more rapidly than the glass encasing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (b) Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95 °F, all six squares are black. When the plastic thermometer is exposed to a temperature of 95 °F, the first liquid crystal square changes color. When the temperature reaches above 96.8 °F, the second liquid crystal square also changes color, and so forth. (c) A firefighter uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. The pyrometer measures infrared radiation (whose emission varies with temperature) from the vent and quickly produces a temperature readout. Infrared thermometers are also frequently used to measure body temperature by gently placing them in the ear canal. Such thermometers are more accurate than the alcohol thermometers placed under the tongue or in the armpit. (credit b: modification of work by Tess Watson; credit c: modification of work by Lamel J. Hinton)

Thermometers measure temperature according to well-defined scales of measurement. The three most common temperature scales are Fahrenheit, Celsius, and Kelvin. Temperature scales are created by identifying two reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

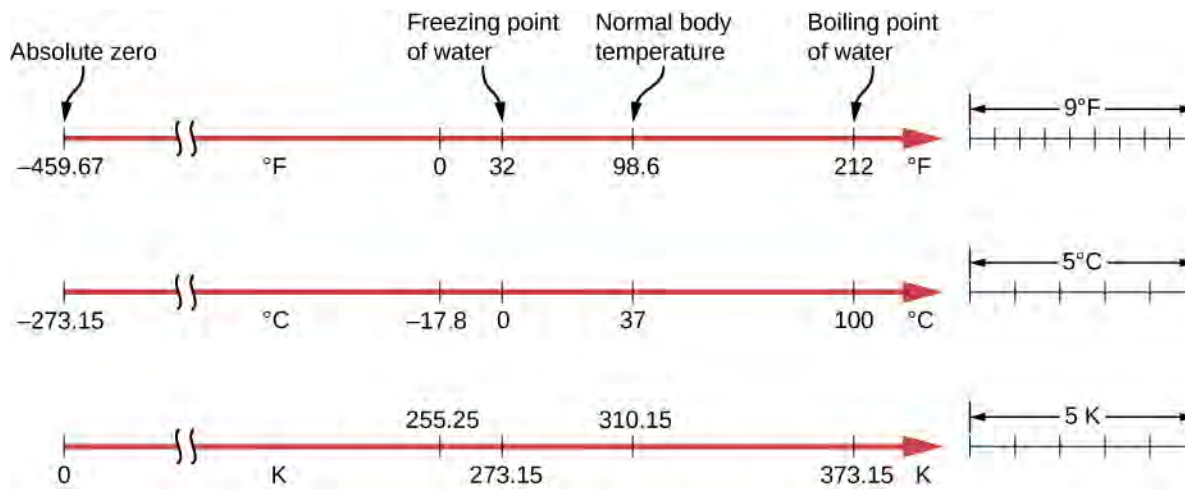
On the **Celsius scale**, the freezing point of water is 0 °C and the boiling point is 100 °C. The unit of temperature on this scale is the **degree Celsius** (°C). The **Fahrenheit scale** (still the most frequently used for common purposes in the United States) has the freezing point of water at 32 °F and the boiling point at 212 °F. Its unit is the **degree Fahrenheit** (°F). You can see that 100 Celsius degrees span the same range as 180 Fahrenheit degrees. Thus, a temperature difference of one degree on the Celsius scale is 1.8 times as large as a difference of one degree on the Fahrenheit scale, or  $\Delta T_F = \frac{9}{5}\Delta T_C$ .

The definition of temperature in terms of molecular motion suggests that there should be a lowest possible temperature, where the average kinetic energy of molecules is zero (or the minimum allowed by quantum mechanics). Experiments confirm the existence of such a temperature, called **absolute zero**. An **absolute temperature scale** is one whose zero point is absolute zero. Such scales are convenient in science because several physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature.

The **Kelvin scale** is the absolute temperature scale that is commonly used in science. The SI temperature unit is the *kelvin*, which is abbreviated K (not accompanied by a degree sign). Thus 0 K is absolute zero. The freezing and boiling points

of water are 273.15 K and 373.15 K, respectively. Therefore, temperature differences are the same in units of kelvins and degrees Celsius, or  $\Delta T_C = \Delta T_K$ .

The relationships between the three common temperature scales are shown in **Figure 1.4**. Temperatures on these scales can be converted using the equations in **Table 1.1**.



**Figure 1.4** Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales are shown. The relative sizes of the scales are also shown.

To convert from...	Use this equation...
Celsius to Fahrenheit	$T_F = \frac{9}{5}T_C + 32$
Fahrenheit to Celsius	$T_C = \frac{5}{9}(T_F - 32)$
Celsius to Kelvin	$T_K = T_C + 273.15$
Kelvin to Celsius	$T_C = T_K - 273.15$
Fahrenheit to Kelvin	$T_K = \frac{5}{9}(T_F - 32) + 273.15$
Kelvin to Fahrenheit	$T_F = \frac{9}{5}(T_K - 273.15) + 32$

**Table 1.1 Temperature Conversions**

To convert between Fahrenheit and Kelvin, convert to Celsius as an intermediate step.

### Example 1.1

#### Converting between Temperature Scales: Room Temperature

“Room temperature” is generally defined in physics to be  $25^\circ\text{C}$ . (a) What is room temperature in  $^\circ\text{F}$ ? (b) What is it in K?

#### Strategy

To answer these questions, all we need to do is choose the correct conversion equations and substitute the known values.

#### Solution

To convert from  $^\circ\text{C}$  to  $^\circ\text{F}$ , use the equation

$$T_F = \frac{9}{5}T_C + 32.$$

Substitute the known value into the equation and solve:

$$T_F = \frac{9}{5}(25^\circ\text{C}) + 32 = 77^\circ\text{F}.$$

Similarly, we find that  $T_K = T_C + 273.15 = 298\text{ K}$ .

The Kelvin scale is part of the SI system of units, so its actual definition is more complicated than the one given above. First, it is not defined in terms of the freezing and boiling points of water, but in terms of the **triple point**. The triple point is the unique combination of temperature and pressure at which ice, liquid water, and water vapor can coexist stably. As will be discussed in the section on phase changes, the coexistence is achieved by lowering the pressure and consequently the boiling point to reach the freezing point. The triple-point temperature is defined as 273.16 K. This definition has the advantage that although the freezing temperature and boiling temperature of water depend on pressure, there is only one triple-point temperature.

Second, even with two points on the scale defined, different thermometers give somewhat different results for other temperatures. Therefore, a standard thermometer is required. Metrologists (experts in the science of measurement) have chosen the *constant-volume gas thermometer* for this purpose. A vessel of constant volume filled with gas is subjected to temperature changes, and the measured temperature is proportional to the change in pressure. Using “TP” to represent the triple point,

$$T = \frac{p}{p_{\text{TP}}}T_{\text{TP}}.$$

The results depend somewhat on the choice of gas, but the less dense the gas in the bulb, the better the results for different gases agree. If the results are extrapolated to zero density, the results agree quite well, with zero pressure corresponding to a temperature of absolute zero.

Constant-volume gas thermometers are big and come to equilibrium slowly, so they are used mostly as standards to calibrate other thermometers.



Visit this [site \(https://openstaxcollege.org//21consvolgasth\)](https://openstaxcollege.org//21consvolgasth) to learn more about the constant-volume gas thermometer.

## 1.3 | Thermal Expansion

### Learning Objectives

By the end of this section, you will be able to:

- Answer qualitative questions about the effects of thermal expansion
- Solve problems involving thermal expansion, including those involving thermal stress

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, which is the change in size or volume of a given system as its temperature changes. The most visible example is the expansion of hot air. When air is heated, it expands and becomes less dense than the surrounding air, which then exerts an (upward) force on the hot air and makes steam and smoke rise, hot air balloons float, and so forth. The same behavior happens in all liquids and gases, driving natural heat transfer upward in homes, oceans, and weather systems, as we will discuss in an upcoming section. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes, as shown in **Figure 1.5**.



**Figure 1.5** (a) Thermal expansion joints like these in the (b) Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: “ŠJû”/Wikimedia Commons)

What is the underlying cause of thermal expansion? As previously mentioned, an increase in temperature means an increase in the kinetic energy of individual atoms. In a solid, unlike in a gas, the molecules are held in place by forces from neighboring molecules; as we saw in **Oscillations** (<http://cnx.org/content/m58360/latest/>), the forces can be modeled as in harmonic springs described by the Lennard-Jones potential. **Energy in Simple Harmonic Motion** ([http://cnx.org/content/m58362/latest/#CNX\\_UPhysics\\_15\\_02\\_LennaJones](http://cnx.org/content/m58362/latest/#CNX_UPhysics_15_02_LennaJones)) shows that such potentials are asymmetrical in that the potential energy increases more steeply when the molecules get closer to each other than when they get farther away. Thus, at a given kinetic energy, the distance moved is greater when neighbors move away from each other than when they move toward each other. The result is that increased kinetic energy (increased temperature) increases the average distance between molecules—the substance expands.

For most substances under ordinary conditions, it is an excellent approximation that there is no preferred direction (that is, the solid is “isotropic”), and an increase in temperature increases the solid’s size by a certain fraction in each dimension. Therefore, if the solid is free to expand or contract, its proportions stay the same; only its overall size changes.

### Linear Thermal Expansion

According to experiments, the dependence of thermal expansion on temperature, substance, and original length is summarized in the equation

$$\frac{dL}{dT} = \alpha L \quad (1.1)$$

where  $L$  is the original length,  $\frac{dL}{dT}$  is the change in length with respect to temperature, and  $\alpha$  is the **coefficient of linear expansion**, a material property that varies slightly with temperature. As  $\alpha$  is nearly constant and also very small, for practical purposes, we use the linear approximation:

$$\Delta L = \alpha L \Delta T. \quad (1.2)$$

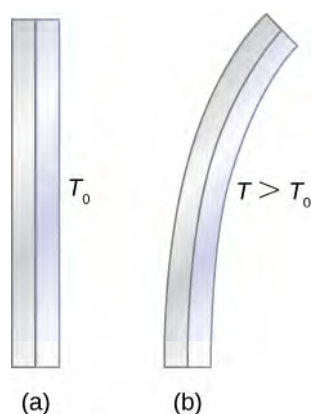
**Table 1.2** lists representative values of the coefficient of linear expansion. As noted earlier,  $\Delta T$  is the same whether it is expressed in units of degrees Celsius or kelvins; thus,  $\alpha$  may have units of  $1/^\circ\text{C}$  or  $1/\text{K}$  with the same value in either case. Approximating  $\alpha$  as a constant is quite accurate for small changes in temperature and sufficient for most practical purposes, even for large changes in temperature. We examine this approximation more closely in the next example.

Material	Coefficient of Linear Expansion $\alpha(1/^\circ\text{C})$	Coefficient of Volume Expansion $\beta(1/^\circ\text{C})$
<i>Solids</i>		
Aluminum	$25 \times 10^{-6}$	$75 \times 10^{-6}$
Brass	$19 \times 10^{-6}$	$56 \times 10^{-6}$
Copper	$17 \times 10^{-6}$	$51 \times 10^{-6}$
Gold	$14 \times 10^{-6}$	$42 \times 10^{-6}$
Iron or steel	$12 \times 10^{-6}$	$35 \times 10^{-6}$
Invar (nickel-iron alloy)	$0.9 \times 10^{-6}$	$2.7 \times 10^{-6}$
Lead	$29 \times 10^{-6}$	$87 \times 10^{-6}$
Silver	$18 \times 10^{-6}$	$54 \times 10^{-6}$
Glass (ordinary)	$9 \times 10^{-6}$	$27 \times 10^{-6}$
Glass (Pyrex®)	$3 \times 10^{-6}$	$9 \times 10^{-6}$
Quartz	$0.4 \times 10^{-6}$	$1 \times 10^{-6}$
Concrete, brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	$2.5 \times 10^{-6}$	$7.5 \times 10^{-6}$
<i>Liquids</i>		
Ether		$1650 \times 10^{-6}$
Ethyl alcohol		$1100 \times 10^{-6}$
Gasoline		$950 \times 10^{-6}$
Glycerin		$500 \times 10^{-6}$
Mercury		$180 \times 10^{-6}$
Water		$210 \times 10^{-6}$
<i>Gases</i>		
Air and most other gases at atmospheric pressure		$3400 \times 10^{-6}$

**Table 1.2 Thermal Expansion Coefficients**

Thermal expansion is exploited in the bimetallic strip (**Figure 1.6**). This device can be used as a thermometer if the curving strip is attached to a pointer on a scale. It can also be used to automatically close or open a switch at a certain temperature, as in older or analog thermostats.





**Figure 1.6** The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right. At a lower temperature, the strip would bend to the left.

## Example 1.2

### Calculating Linear Thermal Expansion

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from  $-15\text{ }^{\circ}\text{C}$  to  $40\text{ }^{\circ}\text{C}$ . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

#### Strategy

Use the equation for linear thermal expansion  $\Delta L = \alpha L \Delta T$  to calculate the change in length,  $\Delta L$ . Use the coefficient of linear expansion  $\alpha$  for steel from **Table 1.2**, and note that the change in temperature  $\Delta T$  is  $55\text{ }^{\circ}\text{C}$ .

#### Solution

Substitute all of the known values into the equation to solve for  $\Delta L$ :

$$\Delta L = \alpha L \Delta T = \left( \frac{12 \times 10^{-6}}{^{\circ}\text{C}} \right) (1275 \text{ m}) (55\text{ }^{\circ}\text{C}) = 0.84 \text{ m}.$$

#### Significance

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

## Thermal Expansion in Two and Three Dimensions

Unconstrained objects expand in all dimensions, as illustrated in **Figure 1.7**. That is, their areas and volumes, as well as their lengths, increase with temperature. Because the proportions stay the same, holes and container volumes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the piece you removed were still in place. The piece would get bigger, so the hole must get bigger too.

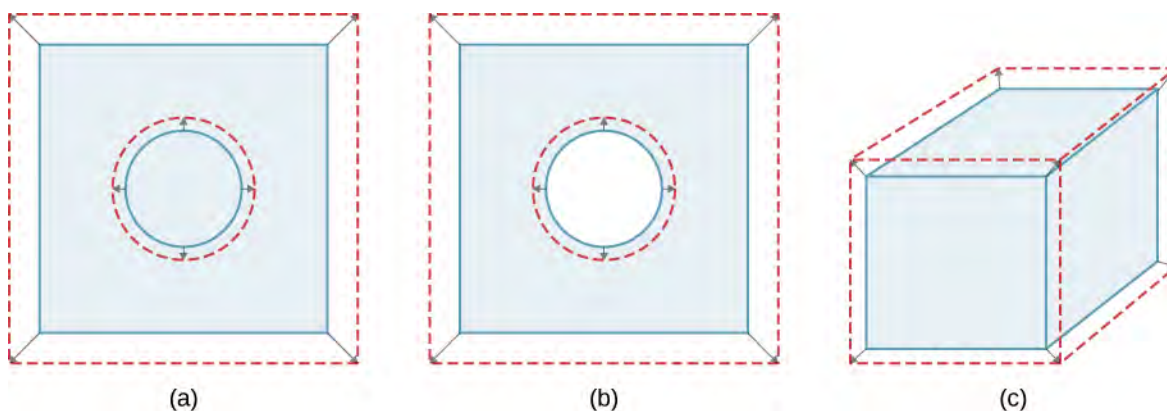
### Thermal Expansion in Two Dimensions

For small temperature changes, the change in area  $\Delta A$  is given by

$$\Delta A = 2\alpha A \Delta T \quad (1.3)$$

where  $\Delta A$  is the change in area  $A$ ,  $\Delta T$  is the change in temperature, and  $\alpha$  is the coefficient of linear expansion,

which varies slightly with temperature.



**Figure 1.7** In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

### Thermal Expansion in Three Dimensions

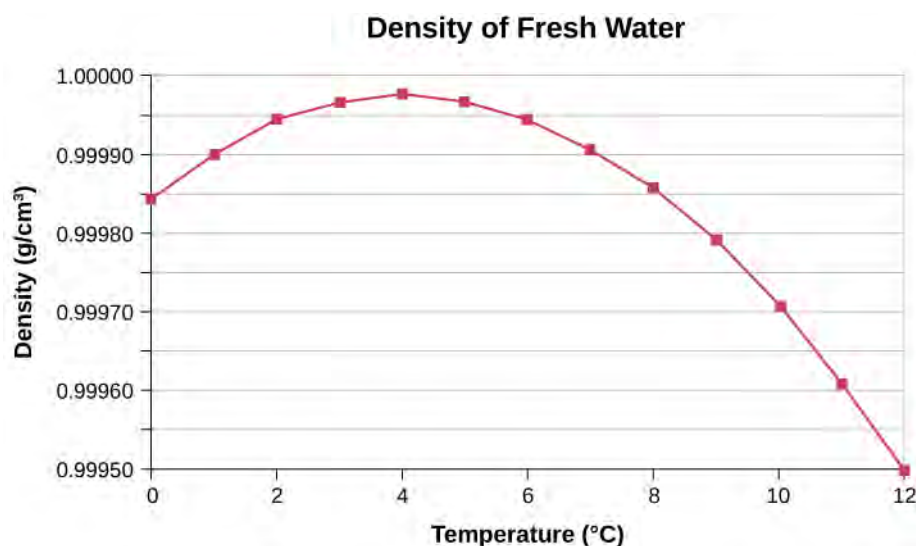
The relationship between volume and temperature  $\frac{dV}{dT}$  is given by  $\frac{dV}{dT} = \beta V \Delta T$ , where  $\beta$  is the **coefficient of volume expansion**. As you can show in **Exercise 1.60**,  $\beta = 3\alpha$ . This equation is usually written as

$$\Delta V = \beta V \Delta T. \quad (1.4)$$

Note that the values of  $\beta$  in **Table 1.2** are equal to  $3\alpha$  except for rounding.

Volume expansion is defined for liquids, but linear and area expansion are not, as a liquid's changes in linear dimensions and area depend on the shape of its container. Thus, **Table 1.2** shows liquids' values of  $\beta$  but not  $\alpha$ .

In general, objects expand with increasing temperature. Water is the most important exception to this rule. Water does expand with increasing temperature (its density *decreases*) at temperatures greater than  $4^\circ\text{C}$  ( $40^\circ\text{F}$ ). However, it is densest at  $+4^\circ\text{C}$  and expands with *decreasing* temperature between  $+4^\circ\text{C}$  and  $0^\circ\text{C}$  ( $40^\circ\text{F}$  to  $32^\circ\text{F}$ ), as shown in **Figure 1.8**. A striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to  $4^\circ\text{C}$ , it is denser than the remaining water and thus sinks to the bottom. This “turnover” leaves a layer of warmer water near the surface, which is then cooled. However, if the temperature in the surface layer drops below  $4^\circ\text{C}$ , that water is less dense than the water below, and thus stays near the top. As a result, the pond surface can freeze over. The layer of ice insulates the liquid water below it from low air temperatures. Fish and other aquatic life can survive in  $4^\circ\text{C}$  water beneath ice, due to this unusual characteristic of water.



**Figure 1.8** This curve shows the density of water as a function of temperature. Note that the thermal expansion at low temperatures is very small. The maximum density at 4 °C is only 0.0075% greater than the density at 2 °C, and 0.012% greater than that at 0 °C. The decrease of density below 4 °C occurs because the liquid water approaches the solid crystal form of ice, which contains more empty space than the liquid.

## Example 1.3

### Calculating Thermal Expansion

Suppose your 60.0-L (15.9 -gal -gal) steel gasoline tank is full of gas that is cool because it has just been pumped from an underground reservoir. Now, both the tank and the gasoline have a temperature of 15.0 °C. How much gasoline has spilled by the time they warm to 35.0 °C?

#### Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank. (The gasoline tank can be treated as solid steel.)

#### Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$\Delta V_s = \beta_s V_s \Delta T.$$

2. The increase in volume of the gasoline is given by this equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T.$$

3. Find the difference in volume to determine the amount spilled as

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_s.$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$\begin{aligned} V_{\text{spill}} &= (\beta_{\text{gas}} - \beta_s) V \Delta T \\ &= [(950 - 35) \times 10^{-6} / ^\circ\text{C}] (60.0 \text{ L}) (20.0 ^\circ\text{C}) \\ &= 1.10 \text{ L}. \end{aligned}$$

#### Significance

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel

expand quickly. The rate of change in thermal properties is discussed later in this chapter.

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist compression with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.



**1.1 Check Your Understanding** Does a given reading on a gasoline gauge indicate more gasoline in cold weather or in hot weather, or does the temperature not matter?

## Thermal Stress

If you change the temperature of an object while preventing it from expanding or contracting, the object is subjected to stress that is compressive if the object would expand in the absence of constraint and tensile if it would contract. This stress resulting from temperature changes is known as **thermal stress**. It can be quite large and can cause damage.

To avoid this stress, engineers may design components so they can expand and contract freely. For instance, in highways, gaps are deliberately left between blocks to prevent thermal stress from developing. When no gaps can be left, engineers must consider thermal stress in their designs. Thus, the reinforcing rods in concrete are made of steel because steel's coefficient of linear expansion is nearly equal to that of concrete.

To calculate the thermal stress in a rod whose ends are both fixed rigidly, we can think of the stress as developing in two steps. First, let the ends be free to expand (or contract) and find the expansion (or contraction). Second, find the stress necessary to compress (or extend) the rod to its original length by the methods you studied in **Static Equilibrium and Elasticity** (<http://cnx.org/content/m58339/latest/>) on static equilibrium and elasticity. In other words, the  $\Delta L$  of the thermal expansion equals the  $\Delta L$  of the elastic distortion (except that the signs are opposite).

### Example 1.4

#### Calculating Thermal Stress

Concrete blocks are laid out next to each other on a highway without any space between them, so they cannot expand. The construction crew did the work on a winter day when the temperature was  $5^\circ\text{C}$ . Find the stress in the blocks on a hot summer day when the temperature is  $38^\circ\text{C}$ . The compressive Young's modulus of concrete is  $Y = 20 \times 10^9 \text{ N/m}^2$ .

#### Strategy

According to the chapter on static equilibrium and elasticity, the stress  $F/A$  is given by

$$\frac{F}{A} = Y \frac{\Delta L}{L_0},$$

where  $Y$  is the Young's modulus of the material—concrete, in this case. In thermal expansion,  $\Delta L = \alpha L_0 \Delta T$ .

We combine these two equations by noting that the two  $\Delta L$ 's are equal, as stated above. Because we are not given  $L_0$  or  $A$ , we can obtain a numerical answer only if they both cancel out.

#### Solution

We substitute the thermal-expansion equation into the elasticity equation to get

$$\frac{F}{A} = Y \frac{\alpha L_0 \Delta T}{L_0} = Y \alpha \Delta T,$$

and as we hoped,  $L_0$  has canceled and  $A$  appears only in  $F/A$ , the notation for the quantity we are calculating.

Now we need only insert the numbers:

$$\frac{F}{A} = (20 \times 10^9 \text{ N/m}^2)(12 \times 10^{-6} /^\circ\text{C})(38^\circ\text{C} - 5^\circ\text{C}) = 7.9 \times 10^6 \text{ N/m}^2.$$

### Significance

The ultimate compressive strength of concrete is  $20 \times 10^6 \text{ N/m}^2$ , so the blocks are unlikely to break. However, the ultimate shear strength of concrete is only  $2 \times 10^6 \text{ N/m}^2$ , so some might chip off.



**1.2 Check Your Understanding** Two objects  $A$  and  $B$  have the same dimensions and are constrained identically.  $A$  is made of a material with a higher thermal expansion coefficient than  $B$ . If the objects are heated identically, will  $A$  feel a greater stress than  $B$ ?

## 1.4 | Heat Transfer, Specific Heat, and Calorimetry

### Learning Objectives

By the end of this section, you will be able to:

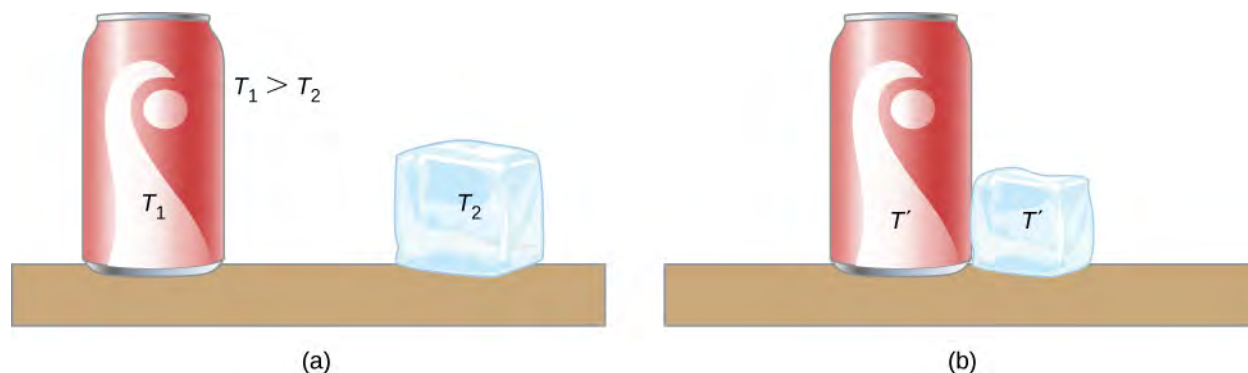
- Explain phenomena involving heat as a form of energy transfer
- Solve problems involving heat transfer

We have seen in previous chapters that energy is one of the fundamental concepts of physics. **Heat** is a type of energy transfer that is caused by a temperature difference, and it can change the temperature of an object. As we learned earlier in this chapter, heat transfer is the movement of energy from one place or material to another as a result of a difference in temperature. Heat transfer is fundamental to such everyday activities as home heating and cooking, as well as many industrial processes. It also forms a basis for the topics in the remainder of this chapter.

We also introduce the concept of internal energy, which can be increased or decreased by heat transfer. We discuss another way to change the internal energy of a system, namely doing work on it. Thus, we are beginning the study of the relationship of heat and work, which is the basis of engines and refrigerators and the central topic (and origin of the name) of thermodynamics.

### Internal Energy and Heat

A thermal system has *internal energy* (also called thermal energy), which is the sum of the mechanical energies of its molecules. A system's internal energy is proportional to its temperature. As we saw earlier in this chapter, if two objects at different temperatures are brought into contact with each other, energy is transferred from the hotter to the colder object until the bodies reach thermal equilibrium (that is, they are at the same temperature). No work is done by either object because no force acts through a distance (as we discussed in **Work and Kinetic Energy** (<http://cnx.org/content/m58307/latest/>)). These observations reveal that heat is energy transferred spontaneously due to a temperature difference. **Figure 1.9** shows an example of heat transfer.



**Figure 1.9** (a) Here, the soft drink has a higher temperature than the ice, so they are not in thermal equilibrium. (b) When the soft drink and ice are allowed to interact, heat is transferred from the drink to the ice due to the difference in temperatures until they reach the same temperature,  $T'$ , achieving equilibrium. In fact, since the soft drink and ice are both in contact with the surrounding air and the bench, the ultimate equilibrium temperature will be the same as that of the surroundings.

The meaning of “heat” in physics is different from its ordinary meaning. For example, in conversation, we may say “the heat was unbearable,” but in physics, we would say that the temperature was high. Heat is a form of energy flow, whereas temperature is not. Incidentally, humans are sensitive to *heat flow* rather than to temperature.

Since heat is a form of energy, its SI unit is the joule (J). Another common unit of energy often used for heat is the **calorie** (cal), defined as the energy needed to change the temperature of 1.00 g of water by 1.00 °C —specifically, between 14.5 °C and 15.5 °C, since there is a slight temperature dependence. Also commonly used is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00 °C. Since mass is most often specified in kilograms, the kilocalorie is convenient. Confusingly, food calories (sometimes called “big calories,” abbreviated Cal) are actually kilocalories, a fact not easily determined from package labeling.

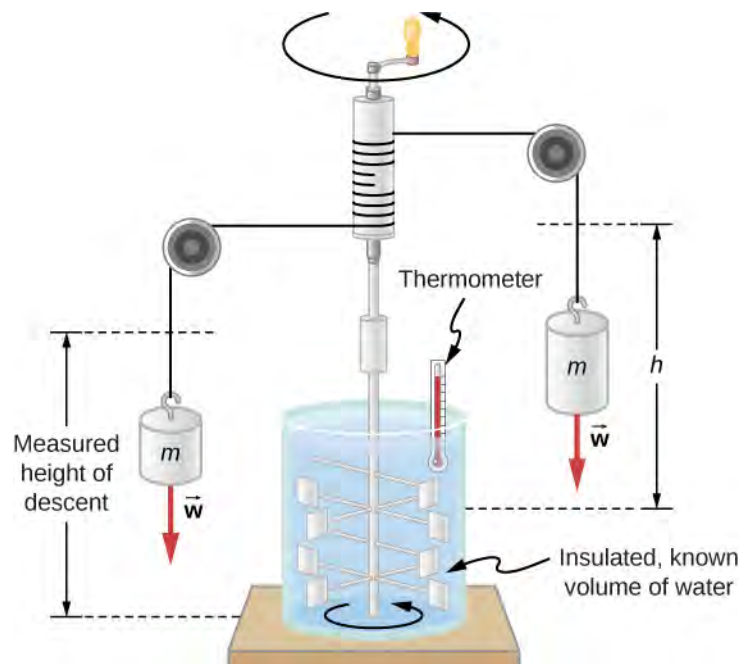
## Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work, which transfers energy into or out of a system. This realization helped establish that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In the units used for these two quantities, the value for this equivalence is

$$1.000 \text{ kcal} = 4186 \text{ J.}$$

We consider this equation to represent the conversion between two units of energy. (Other numbers that you may see refer to calories defined for temperature ranges other than 14.5 °C to 15.5 °C.)

**Figure 1.10** shows one of Joule’s most famous experimental setups for demonstrating that work and heat can produce the same effects and measuring the mechanical equivalent of heat. It helped establish the principle of conservation of energy. Gravitational potential energy ( $U$ ) was converted into kinetic energy ( $K$ ), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. Joule’s contributions to thermodynamics were so significant that the SI unit of energy was named after him.



**Figure 1.10** Joule’s experiment established the equivalence of heat and work. As the masses descended, they caused the paddles to do work,  $W = mgh$ , on the water. The result was a temperature increase,  $\Delta T$ , measured by the thermometer. Joule found that  $\Delta T$  was proportional to  $W$  and thus determined the mechanical equivalent of heat.

Increasing internal energy by heat transfer gives the same result as increasing it by doing work. Therefore, although a system has a well-defined internal energy, we cannot say that it has a certain “heat content” or “work content.” A well-

defined quantity that depends only on the current state of the system, rather than on the history of that system, is known as a *state variable*. Temperature and internal energy are state variables. To sum up this paragraph, *heat and work are not state variables*.

Incidentally, increasing the internal energy of a system does not necessarily increase its temperature. As we'll see in the next section, the temperature does not change when a substance changes from one phase to another. An example is the melting of ice, which can be accomplished by adding heat or by doing frictional work, as when an ice cube is rubbed against a rough surface.

## Temperature Change and Heat Capacity

We have noted that heat transfer often causes temperature change. Experiments show that with no phase change and no work done on or by the system, the transferred heat is typically directly proportional to the change in temperature and to the mass of the system, to a good approximation. (Below we show how to handle situations where the approximation is not valid.) The constant of proportionality depends on the substance and its phase, which may be gas, liquid, or solid. We omit discussion of the fourth phase, plasma, because although it is the most common phase in the universe, it is rare and short-lived on Earth.

We can understand the experimental facts by noting that the transferred heat is the change in the internal energy, which is the total energy of the molecules. Under typical conditions, the total kinetic energy of the molecules  $K_{\text{total}}$  is a constant fraction of the internal energy (for reasons and with exceptions that we'll see in the next chapter). The average kinetic energy of a molecule  $K_{\text{ave}}$  is proportional to the absolute temperature. Therefore, the change in internal energy of a system is typically proportional to the change in temperature and to the number of molecules,  $N$ . Mathematically,  $\Delta U \propto \Delta K_{\text{total}} = NK_{\text{ave}} \propto N\Delta T$ . The dependence on the substance results in large part from the different masses of atoms and molecules. We are considering its heat capacity in terms of its mass, but as we will see in the next chapter, in some cases, heat capacities *per molecule* are similar for different substances. The dependence on substance and phase also results from differences in the potential energy associated with interactions between atoms and molecules.

### Heat Transfer and Temperature Change

A practical approximation for the relationship between heat transfer and temperature change is:

$$Q = mc\Delta T, \quad (1.5)$$

where  $Q$  is the symbol for heat transfer (“quantity of heat”),  $m$  is the mass of the substance, and  $\Delta T$  is the change in temperature. The symbol  $c$  stands for the **specific heat** (also called “*specific heat capacity*”) and depends on the material and phase. The specific heat is numerically equal to the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00 °C. The SI unit for specific heat is J/(kg × K) or J/(kg × °C). (Recall that the temperature change  $\Delta T$  is the same in units of kelvin and degrees Celsius.)

Values of specific heat must generally be measured, because there is no simple way to calculate them precisely. **Table 1.3** lists representative values of specific heat for various substances. We see from this table that the specific heat of water is five times that of glass and 10 times that of iron, which means that it takes five times as much heat to raise the temperature of water a given amount as for glass, and 10 times as much as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

The specific heats of gases depend on what is maintained constant during the heating—typically either the volume or the pressure. In the table, the first specific heat value for each gas is measured at constant volume, and the second (in parentheses) is measured at constant pressure. We will return to this topic in the chapter on the kinetic theory of gases.

Substances	Specific Heat ( $c$ )	
	J/kg · °C	kcal/kg · °C <sup>[2]</sup>
<i>Solids</i>		
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, −50 °C to 0 °C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.40
<i>Liquids</i>		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
<i>Gases<sup>[3]</sup></i>		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100 °C)	1520 (2020)	0.363 (0.482)

**Table 1.3 Specific Heats of Various Substances<sup>[1]</sup>** <sup>[1]</sup>The values for solids and liquids are at constant volume and 25 °C, except as noted. <sup>[2]</sup>These values are identical in units of cal/g · °C. <sup>[3]</sup>Specific heats at constant volume and at 20.0 °C except as noted, and at 1.00 atm pressure. Values in parentheses are specific heats at a constant pressure of 1.00 atm.

In general, specific heat also depends on temperature. Thus, a precise definition of  $c$  for a substance must be given in terms of an infinitesimal change in temperature. To do this, we note that  $c = \frac{1}{m} \frac{\Delta Q}{\Delta T}$  and replace  $\Delta$  with  $d$ :

$$c = \frac{1}{m} \frac{dQ}{dT}.$$

Except for gases, the temperature and volume dependence of the specific heat of most substances is weak at normal



temperatures. Therefore, we will generally take specific heats to be constant at the values given in the table.

## Example 1.5

### Calculating the Required Heat

A 0.500-kg aluminum pan on a stove and 0.250 L of water in it are heated from 20.0 °C to 80.0 °C. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

### Strategy

We can assume that the pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and that of the pan are increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in **Table 1.3**.

### Solution

1. Calculate the temperature difference:

$$\Delta T = T_f - T_i = 60.0 \text{ }^\circ\text{C}.$$

2. Calculate the mass of water. Because the density of water is  $1000 \text{ kg/m}^3$ , 1 L of water has a mass of 1 kg, and the mass of 0.250 L of water is  $m_w = 0.250 \text{ kg}$ .

3. Calculate the heat transferred to the water. Use the specific heat of water in **Table 1.3**:

$$Q_w = m_w c_w \Delta T = (0.250 \text{ kg})(4186 \text{ J/kg }^\circ\text{C})(60.0 \text{ }^\circ\text{C}) = 62.8 \text{ kJ}.$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in **Table 1.3**:

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500 \text{ kg})(900 \text{ J/kg }^\circ\text{C})(60.0 \text{ }^\circ\text{C}) = 27.0 \text{ kJ}.$$

5. Find the total transferred heat:

$$Q_{\text{Total}} = Q_w + Q_{Al} = 89.8 \text{ kJ}.$$

### Significance

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times that of aluminum. Therefore, it takes a bit more than twice as much heat to achieve the given temperature change for the water as for the aluminum pan.

**Example 1.6** illustrates a temperature rise caused by doing work. (The result is the same as if the same amount of energy had been added with a blowtorch instead of mechanically.)

## Example 1.6

### Calculating the Temperature Increase from the Work Done on a Substance

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material (**Figure 1.11**). This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. Since the mass of the truck is much greater than that of the brake material absorbing the energy, the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment; in other words, the brakes may overheat.



**Figure 1.11** The smoking brakes on a braking truck are visible evidence of the mechanical equivalent of heat.

Calculate the temperature increase of 10 kg of brake material with an average specific heat of  $800 \text{ J/kg} \cdot ^\circ\text{C}$  if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

### Strategy

We calculate the gravitational potential energy ( $Mgh$ ) that the entire truck loses in its descent, equate it to the increase in the brakes' internal energy, and then find the temperature increase produced in the brake material alone.

### Solution

First we calculate the change in gravitational potential energy as the truck goes downhill:

$$Mgh = (10,000 \text{ kg})(9.80 \text{ m/s}^2)(75.0 \text{ m}) = 7.35 \times 10^6 \text{ J}.$$

Because the kinetic energy of the truck does not change, conservation of energy tells us the lost potential energy is dissipated, and we assume that 10% of it is transferred to internal energy of the brakes, so take  $Q = Mgh/10$ .

Then we calculate the temperature change from the heat transferred, using

$$\Delta T = \frac{Q}{mc},$$

where  $m$  is the mass of the brake material. Insert the given values to find

$$\Delta T = \frac{7.35 \times 10^5 \text{ J}}{(10 \text{ kg})(800 \text{ J/kg} \cdot ^\circ\text{C})} = 92 \text{ } ^\circ\text{C}.$$

### Significance

If the truck had been traveling for some time, then just before the descent, the brake temperature would probably be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material very high, so this technique is not practical. Instead, the truck would use the technique of engine braking. A different idea underlies the recent technology of hybrid and electric cars, where mechanical energy (kinetic and gravitational potential energy) is converted by the brakes into electrical energy in the battery, a process called regenerative braking.

In a common kind of problem, objects at different temperatures are placed in contact with each other but isolated from everything else, and they are allowed to come into equilibrium. A container that prevents heat transfer in or out is called a **calorimeter**, and the use of a calorimeter to make measurements (typically of heat or specific heat capacity) is called **calorimetry**.

We will use the term “calorimetry problem” to refer to any problem in which the objects concerned are thermally isolated

from their surroundings. An important idea in solving calorimetry problems is that during a heat transfer between objects isolated from their surroundings, the heat gained by the colder object must equal the heat lost by the hotter object, due to conservation of energy:

$$Q_{\text{cold}} + Q_{\text{hot}} = 0. \quad (1.6)$$

We express this idea by writing that the sum of the heats equals zero because the heat gained is usually considered positive; the heat lost, negative.

## Example 1.7

### Calculating the Final Temperature in Calorimetry

Suppose you pour 0.250 kg of 20.0-°C water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of 150 °C. Assume no heat transfer takes place to anything else: The pan is placed on an insulated pad, and heat transfer to the air is neglected in the short time needed to reach equilibrium. Thus, this is a calorimetry problem, even though no isolating container is specified. Also assume that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium?

#### Strategy

Originally, the pan and water are not in thermal equilibrium: The pan is at a higher temperature than the water. Heat transfer restores thermal equilibrium once the water and pan are in contact; it stops once thermal equilibrium between the pan and the water is achieved. The heat lost by the pan is equal to the heat gained by the water—that is the basic principle of calorimetry.

#### Solution

1. Use the equation for heat transfer  $Q = mc\Delta T$  to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}} c_{\text{Al}} (T_f - 150 \text{ }^\circ\text{C}).$$

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water, and the final temperature:

$$Q_{\text{cold}} = m_{\text{w}} c_{\text{w}} (T_f - 20.0 \text{ }^\circ\text{C}).$$

3. Note that  $Q_{\text{hot}} < 0$  and  $Q_{\text{cold}} > 0$  and that as stated above, they must sum to zero:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0 \\ Q_{\text{cold}} &= -Q_{\text{hot}} \\ m_{\text{w}} c_{\text{w}} (T_f - 20.0 \text{ }^\circ\text{C}) &= -m_{\text{Al}} c_{\text{Al}} (T_f - 150 \text{ }^\circ\text{C}). \end{aligned}$$

4. This is a linear equation for the unknown final temperature,  $T_f$ . Solving for  $T_f$ ,

$$T_f = \frac{m_{\text{Al}} c_{\text{Al}} (150 \text{ }^\circ\text{C}) + m_{\text{w}} c_{\text{w}} (20.0 \text{ }^\circ\text{C})}{m_{\text{Al}} c_{\text{Al}} + m_{\text{w}} c_{\text{w}}},$$

and insert the numerical values:

$$T_f = \frac{(0.500 \text{ kg})(900 \text{ J/kg }^\circ\text{C})(150 \text{ }^\circ\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg }^\circ\text{C})(20.0 \text{ }^\circ\text{C})}{(0.500 \text{ kg})(900 \text{ J/kg }^\circ\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg }^\circ\text{C})} = 59.1 \text{ }^\circ\text{C}.$$

#### Significance

Why is the final temperature so much closer to 20.0 °C than to 150 °C? The reason is that water has a greater specific heat than most common substances and thus undergoes a smaller temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature

appreciably. This explains why the temperature of a lake stays relatively constant during the day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).



**1.3 Check Your Understanding** If 25 kJ is necessary to raise the temperature of a rock from 25 °C to 30 °C, how much heat is necessary to heat the rock from 45 °C to 50 °C?

## Example 1.8

### Temperature-Dependent Heat Capacity

At low temperatures, the specific heats of solids are typically proportional to  $T^3$ . The first understanding of this behavior was due to the Dutch physicist Peter Debye, who in 1912, treated atomic oscillations with the quantum theory that Max Planck had recently used for radiation. For instance, a good approximation for the specific heat of salt, NaCl, is  $c = 3.33 \times 10^4 \frac{\text{J}}{\text{kg} \cdot \text{K}} \left( \frac{T}{321 \text{ K}} \right)^3$ . The constant 321 K is called the *Debye temperature* of NaCl,  $\Theta_D$ , and the formula works well when  $T < 0.04\Theta_D$ . Using this formula, how much heat is required to raise the temperature of 24.0 g of NaCl from 5 K to 15 K?

### Solution

Because the heat capacity depends on the temperature, we need to use the equation

$$c = \frac{1}{m} \frac{dQ}{dT}.$$

We solve this equation for  $Q$  by integrating both sides:  $Q = m \int_{T_1}^{T_2} c dT$ .

Then we substitute the given values in and evaluate the integral:

$$Q = (0.024 \text{ kg}) \int_{T_1}^{T_2} 333 \times 10^4 \frac{\text{J}}{\text{kg} \cdot \text{K}} \left( \frac{T}{321 \text{ K}} \right)^3 dT = \left( 6.04 \times 10^{-4} \frac{\text{J}}{\text{K}^4} \right) T^4 \Big|_{5 \text{ K}}^{15 \text{ K}} = 30.2 \text{ J}.$$

### Significance

If we had used the equation  $Q = mc\Delta T$  and the room-temperature specific heat of salt, 880 J/kg · K, we would have gotten a very different value.

## 1.5 | Phase Changes

### Learning Objectives

By the end of this section, you will be able to:

- Describe phase transitions and equilibrium between phases
- Solve problems involving latent heat
- Solve calorimetry problems involving phase changes

Phase transitions play an important theoretical and practical role in the study of heat flow. In melting (or “fusion”), a solid turns into a liquid; the opposite process is freezing. In evaporation, a liquid turns into a gas; the opposite process is condensation.

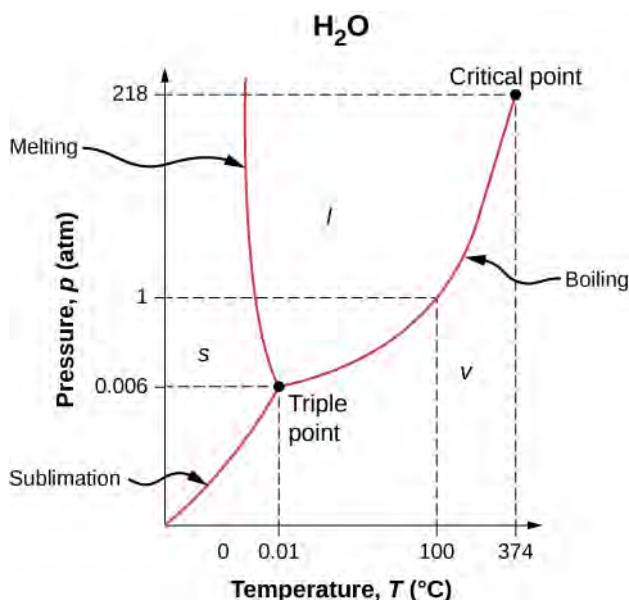
A substance melts or freezes at a temperature called its melting point, and boils (evaporates rapidly) or condenses at its

boiling point. These temperatures depend on pressure. High pressure favors the denser form, so typically, high pressure raises the melting point and boiling point, and low pressure lowers them. For example, the boiling point of water is  $100\text{ }^{\circ}\text{C}$  at  $1.00\text{ atm}$ . At higher pressure, the boiling point is higher, and at lower pressure, it is lower. The main exception is the melting and freezing of water, discussed in the next section.

## Phase Diagrams

The phase of a given substance depends on the pressure and temperature. Thus, plots of pressure versus temperature showing the phase in each region provide considerable insight into thermal properties of substances. Such a  $pT$  graph is called a **phase diagram**.

**Figure 1.12** shows the phase diagram for water. Using the graph, if you know the pressure and temperature, you can determine the phase of water. The solid curves—boundaries between phases—indicate phase transitions, that is, temperatures and pressures at which the phases coexist. For example, the boiling point of water is  $100\text{ }^{\circ}\text{C}$  at  $1.00\text{ atm}$ . As the pressure increases, the boiling temperature rises gradually to  $374\text{ }^{\circ}\text{C}$  at a pressure of  $218\text{ atm}$ . A pressure cooker (or even a covered pot) cooks food faster than an open pot, because the water can exist as a liquid at temperatures greater than  $100\text{ }^{\circ}\text{C}$  without all boiling away. (As we'll see in the next section, liquid water conducts heat better than steam or hot air.) The boiling point curve ends at a certain point called the **critical point**—that is, a **critical temperature**, above which the liquid and gas phases cannot be distinguished; the substance is called a *supercritical fluid*. At sufficiently high pressure above the critical point, the gas has the density of a liquid but does not condense. Carbon dioxide, for example, is supercritical at all temperatures above  $31.0\text{ }^{\circ}\text{C}$ . **Critical pressure** is the pressure of the critical point.



**Figure 1.12** The phase diagram ( $pT$  graph) for water shows solid (s), liquid (l), and vapor (v) phases. At temperatures and pressure above those of the critical point, there is no distinction between liquid and vapor. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—it omits several exotic phases of ice at higher pressures. The phase diagram of water is unusual because the melting-point curve has a negative slope, showing that you can melt ice by *increasing* the pressure.

Similarly, the curve between the solid and liquid regions in **Figure 1.12** gives the melting temperature at various pressures. For example, the melting point is  $0\text{ }^{\circ}\text{C}$  at  $1.00\text{ atm}$ , as expected. Water has the unusual property that ice is less dense than liquid water at the melting point, so at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. That is, the melting temperature of ice falls with increased pressure, as the phase diagram shows. For example, when a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards, the water refreezes and forms an ice layer.

As you learned in the earlier section on thermometers and temperature scales, the triple point is the combination of

temperature and pressure at which ice, liquid water, and water vapor can coexist stably—that is, all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K (0.01 °C) and 611.2 Pa; that is a more accurate calibration temperature than the melting point of water at 1.00 atm, or 273.15 K (0.0 °C).

 View this [video \(https://openstaxcollege.org//21triplepoint\)](https://openstaxcollege.org//21triplepoint) to see a substance at its triple point.

At pressures below that of the triple point, there is no liquid phase; the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm. The phase change from solid to gas is called **sublimation**. You may have noticed that snow can disappear into thin air without a trace of liquid water, or that ice cubes can disappear in a freezer. Both are examples of sublimation. The reverse also happens: Frost can form on very cold windows without going through the liquid stage. **Figure 1.13** shows the result, as well as showing a familiar example of sublimation. Carbon dioxide has no liquid phase at atmospheric pressure. Solid  $\text{CO}_2$  is known as dry ice because instead of melting, it sublimates. Its sublimation temperature at atmospheric pressure is  $-78\text{ }^\circ\text{C}$ . Certain air fresheners use the sublimation of a solid to spread a perfume around a room. Some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

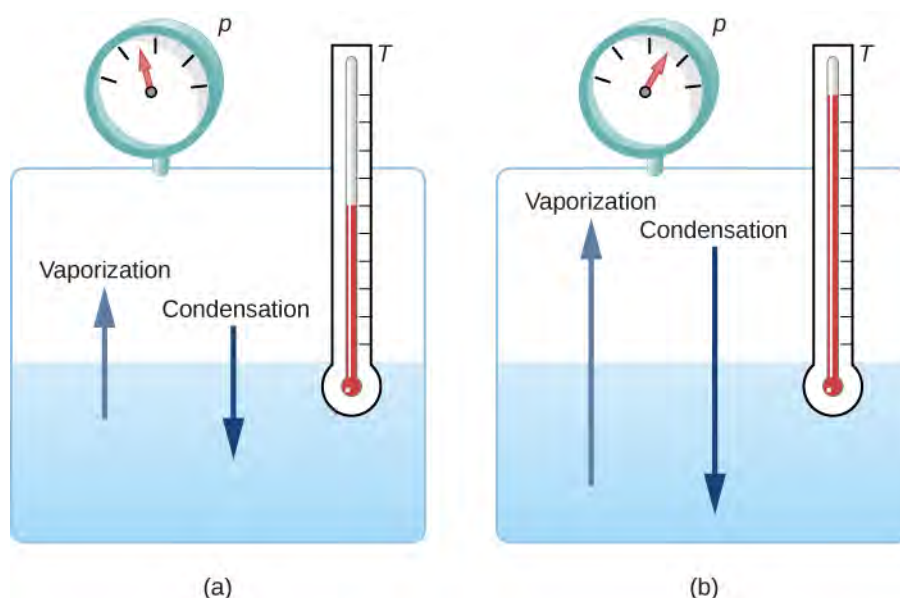


**Figure 1.13** Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible “smoke” consists of water droplets that condensed in the air cooled by the dry ice. (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit a: modification of work by Windell Oskay; credit b: modification of work by Liz West)

## Equilibrium


At the melting temperature, the solid and liquid phases are in equilibrium. If heat is added, some of the solid will melt, and if heat is removed, some of the liquid will freeze. The situation is somewhat more complex for liquid-gas equilibrium. Generally, liquid and gas are in equilibrium at any temperature. We call the gas phase a **vapor** when it exists at a temperature below the boiling temperature, as it does for water at  $20.0\text{ }^\circ\text{C}$ . Liquid in a closed container at a fixed temperature evaporates until the pressure of the gas reaches a certain value, called the **vapor pressure**, which depends on the gas and the temperature. At this equilibrium, if heat is added, some of the liquid will evaporate, and if heat is removed, some of the gas will condense; molecules either join the liquid or form suspended droplets. If there is not enough liquid for the gas to reach the vapor pressure in the container, all the liquid eventually evaporates.

If the vapor pressure of the liquid is greater than the *total* ambient pressure, including that of any air (or other gas), the liquid evaporates rapidly; in other words, it boils. Thus, the boiling point of a liquid at a given pressure is the temperature at which its vapor pressure equals the ambient pressure. Liquid and gas phases are in equilibrium at the boiling temperature (**Figure 1.14**). If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their amounts.



**Figure 1.14** Equilibrium between liquid and gas at two different boiling points inside a closed container. (a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster, that is, the rate at which molecules leave the liquid and enter the gas is faster. This increases the number of molecules in the gas, which increases the gas pressure, which in turn increases the rate at which gas molecules condense and enter the liquid. The pressure stops increasing when it reaches the point where the boiling rate and the condensation rate are equal. The gas and liquid are in equilibrium again at this higher temperature and pressure.

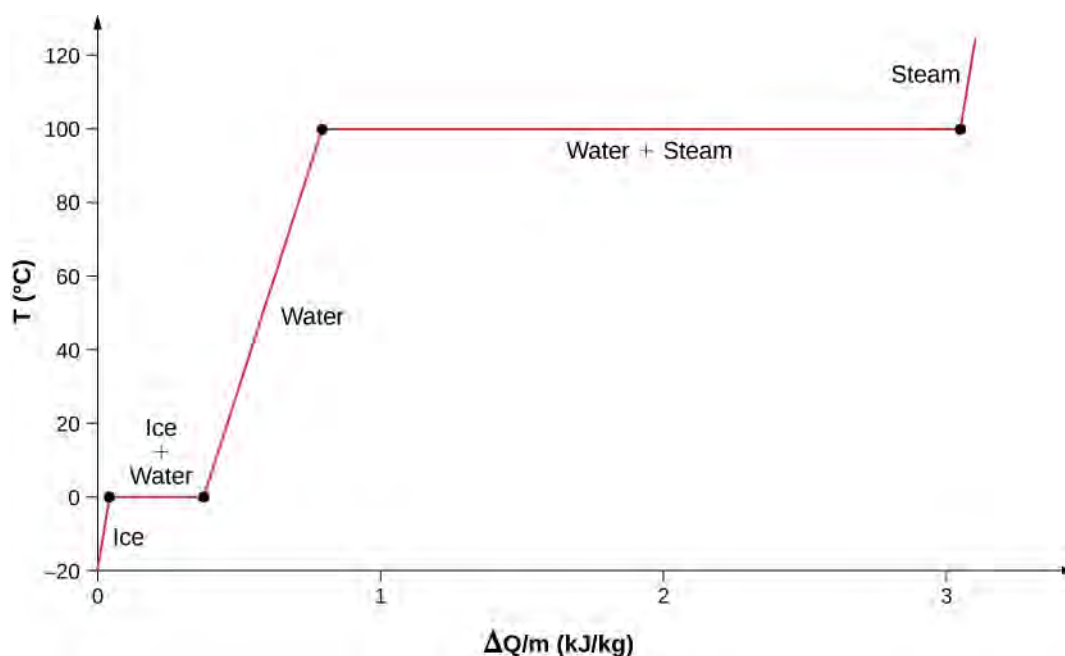
For water,  $100\text{ }^{\circ}\text{C}$  is the boiling point at  $1.00\text{ atm}$ , so water and steam should exist in equilibrium under these conditions. Why does an open pot of water at  $100\text{ }^{\circ}\text{C}$  boil completely away? The gas surrounding an open pot is not pure water: it is mixed with air. If pure water and steam are in a closed container at  $100\text{ }^{\circ}\text{C}$  and  $1.00\text{ atm}$ , they will coexist—but with air over the pot, there are fewer water molecules to condense, and water boils away. Another way to see this is that at the boiling point, the vapor pressure equals the ambient pressure. However, part of the ambient pressure is due to air, so the pressure of the steam is less than the vapor pressure at that temperature, and evaporation continues. Incidentally, the equilibrium vapor pressure of solids is not zero, a fact that accounts for sublimation.

 **1.4 Check Your Understanding** Explain why a cup of water (or soda) with ice cubes stays at  $0\text{ }^{\circ}\text{C}$ , even on a hot summer day.

## Phase Change and Latent Heat

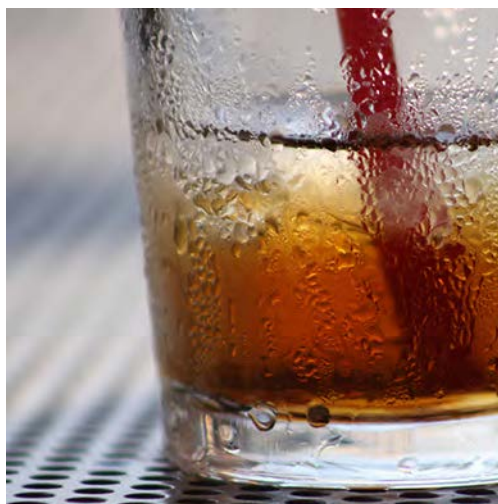
So far, we have discussed heat transfers that cause temperature change. However, in a phase transition, heat transfer does not cause any temperature change.

For an example of phase changes, consider the addition of heat to a sample of ice at  $-20\text{ }^{\circ}\text{C}$  (**Figure 1.15**) and atmospheric pressure. The temperature of the ice rises linearly, absorbing heat at a constant rate of  $2090\text{ J/kg}\cdot^{\circ}\text{C}$  until it reaches  $0\text{ }^{\circ}\text{C}$ . Once at this temperature, the ice begins to melt and continues until it has all melted, absorbing  $333\text{ kJ/kg}$  of heat. The temperature remains constant at  $0\text{ }^{\circ}\text{C}$  during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of  $4186\text{ J/kg}\cdot^{\circ}\text{C}$ . At  $100\text{ }^{\circ}\text{C}$ , the water begins to boil. The temperature again remains constant during this phase change while the water absorbs  $2256\text{ kJ/kg}$  of heat and turns into steam. When all the liquid has become steam, the temperature rises again, absorbing heat at a rate of  $2020\text{ J/kg}\cdot^{\circ}\text{C}$ . If we started with steam and cooled it to make it condense into liquid water and freeze into ice, the process would exactly reverse, with the temperature again constant during each phase transition.



**Figure 1.15** Temperature versus heat. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in the system. The long stretches of constant temperatures at 0 °C and 100 °C reflect the large amounts of heat needed to cause melting and vaporization, respectively.

Where does the heat added during melting or boiling go, considering that the temperature does not change until the transition is complete? Energy is required to melt a solid, because the attractive forces between the molecules in the solid must be broken apart, so that in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Energy is needed to vaporize a liquid for similar reasons. Conversely, work is done by attractive forces when molecules are brought together during freezing and condensation. That energy must be transferred out of the system, usually in the form of heat, to allow the molecules to stay together (**Figure 1.18**). Thus, condensation occurs in association with cold objects—the glass in **Figure 1.16**, for example.



**Figure 1.16** Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced. The air cannot hold as much water as it did at room temperature, so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)



The energy released when a liquid freezes is used by orange growers when the temperature approaches  $0^{\circ}\text{C}$ . Growers spray water on the trees so that the water freezes and heat is released to the growing oranges. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit (**Figure 1.17**).



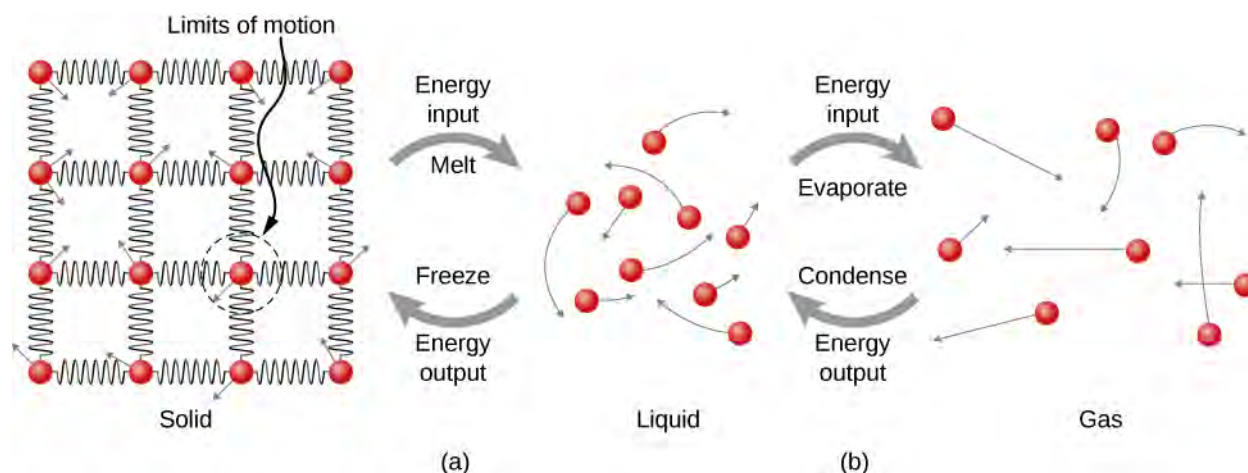
**Figure 1.17** The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below  $0^{\circ}\text{C}$ . Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

The energy involved in a phase change depends on the number of bonds or force pairs and their strength. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The energy per unit mass required to change a substance from the solid phase to the liquid phase, or released when the substance changes from liquid to solid, is known as the **heat of fusion**. The energy per unit mass required to change a substance from the liquid phase to the vapor phase is known as the **heat of vaporization**. The strength of the forces depends on the type of molecules. The heat  $Q$  absorbed or released in a phase change in a sample of mass  $m$  is given by

$$Q = mL_f(\text{melting/freezing}) \quad (1.7)$$

$$Q = mL_v(\text{vaporization/condensation}) \quad (1.8)$$

where the latent heat of fusion  $L_f$  and latent heat of vaporization  $L_v$  are material constants that are determined experimentally. (Latent heats are also called **latent heat coefficients** and heats of transformation.) These constants are “latent,” or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system, so in effect, the energy is hidden.



**Figure 1.18** (a) Energy is required to partially overcome the attractive forces (modeled as springs) between molecules in a solid to form a liquid. That same energy must be removed from the liquid for freezing to take place. (b) Molecules become separated by large distances when going from liquid to vapor, requiring significant energy to completely overcome molecular attraction. The same energy must be removed from the vapor for condensation to take place.

**Table 1.4** lists representative values of  $L_f$  and  $L_v$  in kJ/kg, together with melting and boiling points. Note that in general,  $L_v > L_f$ . The table shows that the amounts of energy involved in phase changes can easily be comparable to or greater than those involved in temperature changes, as **Figure 1.15** and the accompanying discussion also showed.

Substance	Melting Point (°C)	$L_f$		Boiling Point (°C)	$L_v$	
		kJ/kg	kcal/kg		kJ/kg	kcal/kg
Helium <sup>[2]</sup>	-272.2 (0.95 K)	5.23	1.25	-268.9(4.2 K)	20.9	4.99
Hydrogen	-259.3(13.9 K)	58.6	14.0	-252.9(20.2 K)	452	108
Nitrogen	-210.0(63.2 K)	25.5	6.09	-195.8(77.4 K)	201	48.0
Oxygen	-218.8(54.4 K)	13.8	3.30	-183.0(90.2 K)	213	50.9
Ethanol	-114	104	24.9	78.3	854	204
Ammonia	-75	332	79.3	-33.4	1370	327
Mercury	-38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256 <sup>[3]</sup>	539 <sup>[4]</sup>
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558

**Table 1.4 Heats of Fusion and Vaporization<sup>[1]</sup>** <sup>[1]</sup>Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm). <sup>[2]</sup>Helium has no solid phase at atmospheric pressure. The melting point given is at a pressure of 2.5 MPa. <sup>[3]</sup>At 37.0 °C (body temperature), the heat of vaporization  $L_v$  for water is 2430 kJ/kg or 580 kcal/kg. <sup>[4]</sup>At 37.0 °C (body temperature), the heat of vaporization,  $L_v$  for water is 2430 kJ/kg or 580 kcal/kg.

	$L_f$			$L_v$		
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

**Table 1.4 Heats of Fusion and Vaporization**<sup>[1]</sup> <sup>[1]</sup>Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm). <sup>[2]</sup>Helium has no solid phase at atmospheric pressure. The melting point given is at a pressure of 2.5 MPa. <sup>[3]</sup>At 37.0 °C (body temperature), the heat of vaporization  $L_v$  for water is 2430 kJ/kg or 580 kcal/kg. <sup>[4]</sup>At 37.0 °C (body temperature), the heat of vaporization,  $L_v$  for water is 2430 kJ/kg or 580 kcal/kg.

Phase changes can have a strong stabilizing effect on temperatures that are not near the melting and boiling points, since evaporation and condensation occur even at temperatures below the boiling point. For example, air temperatures in humid climates rarely go above approximately 38.0 °C because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point—the temperature where condensation occurs given the concentration of water vapor in the air—because so much heat is released when water vapor condenses.

More energy is required to evaporate water below the boiling point than at the boiling point, because the kinetic energy of water molecules at temperatures below 100 °C is less than that at 100 °C, so less energy is available from random thermal motions. For example, at body temperature, evaporation of sweat from the skin requires a heat input of 2428 kJ/kg, which is about 10% higher than the latent heat of vaporization at 100 °C. This heat comes from the skin, and this evaporative cooling effect of sweating helps reduce the body temperature in hot weather. However, high humidity inhibits evaporation, so that body temperature might rise, while unevaporated sweat might be left on your brow.

## Example 1.9

### Calculating Final Temperature from Phase Change

Three ice cubes are used to chill a soda at 20 °C with mass  $m_{\text{soda}} = 0.25 \text{ kg}$ . The ice is at 0 °C and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored and that the soda has the same specific heat as water. Find the final temperature when all ice has melted.

#### Strategy

The ice cubes are at the melting temperature of 0 °C. Heat is transferred from the soda to the ice for melting. Melting yields water at 0 °C, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium.

The heat transferred to the ice is

$$Q_{\text{ice}} = m_{\text{ice}} L_f + m_{\text{ice}} c_w (T_f - 0 \text{ }^\circ\text{C}).$$

The heat given off by the soda is

$$Q_{\text{soda}} = m_{\text{soda}} c_w (T_f - 20 \text{ }^\circ\text{C}).$$

Since no heat is lost,  $Q_{\text{ice}} = -Q_{\text{soda}}$ , as in **Example 1.7**, so that

$$m_{\text{ice}} L_f + m_{\text{ice}} c_w (T_f - 0 \text{ }^\circ\text{C}) = -m_{\text{soda}} c_w (T_f - 20 \text{ }^\circ\text{C}).$$

Solve for the unknown quantity  $T_f$ :

$$T_f = \frac{m_{\text{soda}} c_w (20 \text{ }^\circ\text{C}) - m_{\text{ice}} L_f}{(m_{\text{soda}} + m_{\text{ice}}) c_w}.$$

### Solution

First we identify the known quantities. The mass of ice is  $m_{\text{ice}} = 3 \times 6.0 \text{ g} = 0.018 \text{ kg}$  and the mass of soda is  $m_{\text{soda}} = 0.25 \text{ kg}$ . Then we calculate the final temperature:

$$T_f = \frac{20,930 \text{ J} - 6012 \text{ J}}{1122 \text{ J/}^\circ\text{C}} = 13 \text{ }^\circ\text{C}.$$

### Significance

This example illustrates the large energies involved during a phase change. The mass of ice is about 7% of the mass of the soda but leads to a noticeable change in the temperature of the soda. Although we assumed that the ice was at the freezing temperature, this is unrealistic for ice straight out of a freezer: The typical temperature is  $-6 \text{ }^\circ\text{C}$ . However, this correction makes no significant change from the result we found. Can you explain why?

Like solid-liquid and liquid-vapor transitions, direct solid-vapor transitions or sublimations involve heat. The energy transferred is given by the equation  $Q = mL_s$ , where  $L_s$  is the **heat of sublimation**, analogous to  $L_f$  and  $L_v$ . The heat of sublimation at a given temperature is equal to the heat of fusion plus the heat of vaporization at that temperature.

We can now calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place. Keep in mind that heat transfer and work can cause both temperature and phase changes.

### Problem-Solving Strategy: The Effects of Heat Transfer

1. Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system? When it is not obvious whether a phase change occurs or not, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. Identify and list all objects that change temperature or phase.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
4. Make a list of what is given or what can be inferred from the problem as stated (identify the knowns). If there is a temperature change, the transferred heat depends on the specific heat of the substance (**Heat Transfer, Specific Heat, and Calorimetry**), and if there is a phase change, the transferred heat depends on the latent heat of the substance (**Table 1.4**).
5. Solve the appropriate equation for the quantity to be determined (the unknown).
6. Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units. You may need to do this in steps if there is more than one state to the process, such as a temperature change followed by a phase change. However, in a calorimetry problem, each step corresponds to a term in the single equation  $Q_{\text{hot}} + Q_{\text{cold}} = 0$ .
7. Check the answer to see if it is reasonable. Does it make sense? As an example, be certain that any temperature change does not also cause a phase change that you have not taken into account.



**1.5 Check Your Understanding** Why does snow often remain even when daytime temperatures are higher than the freezing temperature?

## 1.6 | Mechanisms of Heat Transfer

### Learning Objectives

By the end of this section, you will be able to:

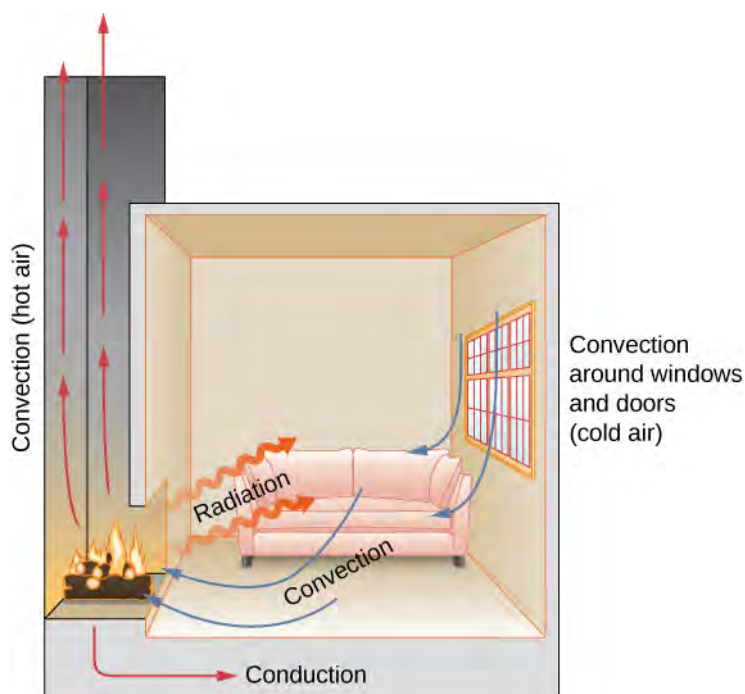
- Explain some phenomena that involve conductive, convective, and radiative heat transfer
- Solve problems on the relationships between heat transfer, time, and rate of heat transfer
- Solve problems using the formulas for conduction and radiation

Just as interesting as the effects of heat transfer on a system are the methods by which it occurs. Whenever there is a temperature difference, heat transfer occurs. It may occur rapidly, as through a cooking pan, or slowly, as through the walls of a picnic ice chest. So many processes involve heat transfer that it is hard to imagine a situation where no heat transfer occurs. Yet every heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know that thermal motion of the atoms and molecules occurs at any temperature above absolute zero.) Heat transferred from the burner of a stove through the bottom of a pan to food in the pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of Earth by the Sun. A less obvious example is thermal radiation from the human body.

In the illustration at the beginning of this chapter, the fire warms the snowshoers' faces largely by radiation. Convection carries some heat to them, but most of the air flow from the fire is upward (creating the familiar shape of flames), carrying heat to the food being cooked and into the sky. The snowshoers wear clothes designed with low conductivity to prevent heat flow out of their bodies.

In this section, we examine these methods in some detail. Each method has unique and interesting characteristics, but all three have two things in common: They transfer heat solely because of a temperature difference, and the greater the temperature difference, the faster the heat transfer (**Figure 1.19**).



**Figure 1.19** In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but much slower. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.



**1.6 Check Your Understanding** Name an example from daily life (different from the text) for each mechanism of heat transfer.

## Conduction

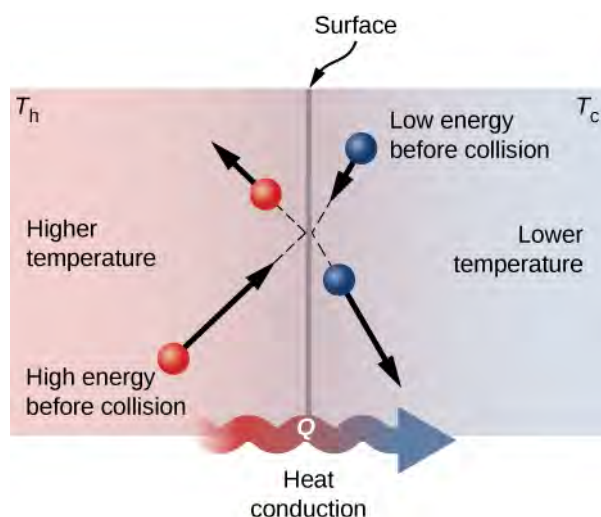
As you walk barefoot across the living room carpet in a cold house and then step onto the kitchen tile floor, your feet feel colder on the tile. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation is explained by the different rates of heat transfer: The heat loss is faster for skin in contact with the tiles than with the carpet, so the sensation of cold is more intense.

Some materials conduct thermal energy faster than others. **Figure 1.20** shows a material that conducts heat slowly—it is a good thermal insulator, or poor heat conductor—used to reduce heat flow into and out of a house.



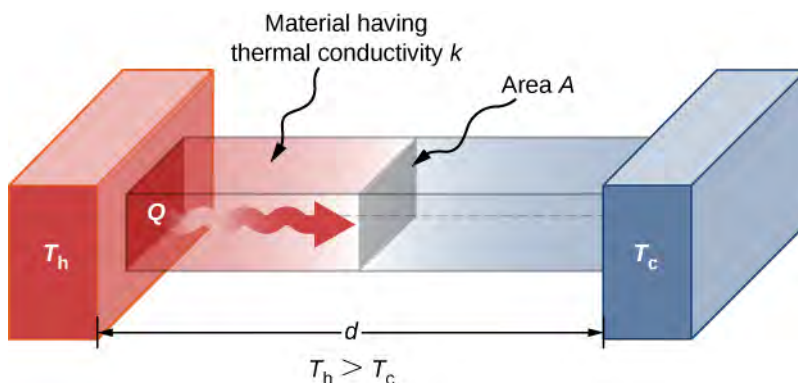
**Figure 1.20** Insulation is used to limit the conduction of heat from the inside to the outside (in winter) and from the outside to the inside (in summer). (credit: Giles Douglas)

A molecular picture of heat conduction will help justify the equation that describes it. **Figure 1.21** shows molecules in two bodies at different temperatures,  $T_h$  and  $T_c$ , for “hot” and “cold.” The average kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, energy transfers from the high-energy to the low-energy molecule. In a metal, the picture would also include free valence electrons colliding with each other and with atoms, likewise transferring energy. The cumulative effect of all collisions is a net flux of heat from the hotter body to the colder body. Thus, the rate of heat transfer increases with increasing temperature difference  $\Delta T = T_h - T_c$ . If the temperatures are the same, the net heat transfer rate is zero. Because the number of collisions increases with increasing area, heat conduction is proportional to the cross-sectional area—a second factor in the equation.



**Figure 1.21** Molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower-temperature region (right side) has low energy before collision, but its energy increases after colliding with a high-energy molecule at the contact surface. In contrast, a molecule in the higher-temperature region (left side) has high energy before collision, but its energy decreases after colliding with a low-energy molecule at the contact surface.

A third quantity that affects the conduction rate is the thickness of the material through which heat transfers. **Figure 1.22** shows a slab of material with a higher temperature on the left than on the right. Heat transfers from the left to the right by a series of molecular collisions. The greater the distance between hot and cold, the more time the material takes to transfer the same amount of heat.



**Figure 1.22** Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber.

All four of these quantities appear in a simple equation deduced from and confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in **Figure 1.22**, is given by

$$P = \frac{dQ}{dT} = \frac{kA(T_h - T_c)}{d} \quad (1.9)$$



where  $P$  is the power or rate of heat transfer in watts or in kilocalories per second,  $A$  and  $d$  are its surface area and thickness, as shown in **Figure 1.22**,  $T_h - T_c$  is the temperature difference across the slab, and  $k$  is the **thermal conductivity** of the material. **Table 1.5** gives representative values of thermal conductivity.

More generally, we can write

$$P = -kA \frac{dT}{dx},$$

where  $x$  is the coordinate in the direction of heat flow. Since in **Figure 1.22**, the power and area are constant,  $dT/dx$  is constant, and the temperature decreases linearly from  $T_h$  to  $T_c$ .

Substance	Thermal Conductivity $k$ (W/m·°C)
Diamond	2000
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Polystyrene foam	0.010

**Table 1.5 Thermal Conductivities of Common Substances** Values are given for temperatures near 0 °C .

## Example 1.10

### Calculating Heat Transfer through Conduction

A polystyrene foam icebox has a total area of  $0.950 \text{ m}^2$  and walls with an average thickness of 2.50 cm. The box contains ice, water, and canned beverages at  $0 \text{ }^\circ\text{C}$ . The inside of the box is kept cold by melting ice. How much ice melts in one day if the icebox is kept in the trunk of a car at  $35.0 \text{ }^\circ\text{C}$  ?

### Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

### Solution

First we identify the knowns.

$k = 0.010 \text{ W/m} \cdot ^\circ\text{C}$  for polystyrene foam;  $A = 0.950 \text{ m}^2$ ;  $d = 2.50 \text{ cm} = 0.0250 \text{ m}$ ;  $T_c = 0^\circ\text{C}$ ;  $T_h = 35.0^\circ\text{C}$ ;  $t = 1 \text{ day} = 24 \text{ hours} = 86,400 \text{ s}$ .

Then we identify the unknowns. We need to solve for the mass of the ice,  $m$ . We also need to solve for the net heat transferred to melt the ice,  $Q$ . The rate of heat transfer by conduction is given by

$$P = \frac{dQ}{dT} = \frac{kA(T_h - T_c)}{d}.$$

The heat used to melt the ice is  $Q = mL_f$ . We insert the known values:

$$P = \frac{(0.010 \text{ W/m} \cdot ^\circ\text{C})(0.950 \text{ m}^2)(35.0^\circ\text{C} - 0^\circ\text{C})}{0.0250 \text{ m}} = 13.3 \text{ W}.$$

Multiplying the rate of heat transfer by the time (1 day = 86,400 s), we obtain

$$Q = Pt = (13.3 \text{ W})(86,400 \text{ s}) = 1.15 \times 10^6 \text{ J}.$$

We set this equal to the heat transferred to melt the ice,  $Q = mL_f$ , and solve for the mass  $m$ :

$$m = \frac{Q}{L_f} = \frac{1.15 \times 10^6 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 3.44 \text{ kg}.$$

### Significance

The result of 3.44 kg, or about 7.6 lb, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

**Table 1.5** shows that polystyrene foam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like polystyrene foam, these all contain many small pockets of air, taking advantage of air's poor thermal conductivity.

In developing insulation, the smaller the conductivity  $k$  and the larger the thickness  $d$ , the better. Thus, the ratio  $d/k$ , called the *R factor*, is large for a good insulator. The rate of conductive heat transfer is inversely proportional to  $R$ .  $R$  factors are most commonly quoted for household insulation, refrigerators, and the like. Unfortunately, in the United States,  $R$  is still in non-metric units of  $\text{ft}^2 \cdot ^\circ\text{F} \cdot \text{h/Btu}$ , although the unit usually goes unstated [1 British thermal unit (Btu) is the amount of energy needed to change the temperature of 1.0 lb of water by  $1.0^\circ\text{F}$ , which is 1055.1 J]. A couple of representative values are an  $R$  factor of 11 for 3.5-inch-thick fiberglass batts (pieces) of insulation and an  $R$  factor of 19 for 6.5-inch-thick fiberglass batts (**Figure 1.23**). In the US, walls are usually insulated with 3.5-inch batts, whereas ceilings are usually insulated with 6.5-inch batts. In cold climates, thicker batts may be used.



**Figure 1.23** The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment. (credit: Tracey Nicholls)

Note that in **Table 1.5**, most of the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, because they contain many free electrons that can transport thermal energy. (Diamond, an electrical insulator, conducts heat by atomic vibrations.) Cooking utensils are typically made from good conductors, but the handles of those used on the stove are made from good insulators (bad conductors).

## Example 1.11

### Two Conductors End to End

A steel rod and an aluminum rod, each of diameter 1.00 cm and length 25.0 cm, are welded end to end. One end of the steel rod is placed in a large tank of boiling water at  $100\text{ }^{\circ}\text{C}$ , while the far end of the aluminum rod is placed in a large tank of water at  $20\text{ }^{\circ}\text{C}$ . The rods are insulated so that no heat escapes from their surfaces. What is the temperature at the joint, and what is the rate of heat conduction through this composite rod?

#### Strategy

The heat that enters the steel rod from the boiling water has no place to go but through the steel rod, then through the aluminum rod, to the cold water. Therefore, we can equate the rate of conduction through the steel to the rate of conduction through the aluminum.

We repeat the calculation with a second method, in which we use the thermal resistance  $R$  of the rod, since it simply adds when two rods are joined end to end. (We will use a similar method in the chapter on direct-current circuits.)

#### Solution

1. Identify the knowns and convert them to SI units.

The length of each rod is  $L_{\text{Al}} = L_{\text{steel}} = 0.25\text{ m}$ , the cross-sectional area of each rod is

$A_{\text{Al}} = A_{\text{steel}} = 7.85 \times 10^{-5}\text{ m}^2$ , the thermal conductivity of aluminum is  $k_{\text{Al}} = 220\text{ W/m}\cdot^{\circ}\text{C}$ , the

thermal conductivity of steel is  $k_{\text{steel}} = 80\text{ W/m}\cdot^{\circ}\text{C}$ , the temperature at the hot end is  $T = 100\text{ }^{\circ}\text{C}$ , and

the temperature at the cold end is  $T = 20\text{ }^{\circ}\text{C}$ .

2. Calculate the heat-conduction rate through the steel rod and the heat-conduction rate through the aluminum rod in terms of the unknown temperature  $T$  at the joint:

$$\begin{aligned} P_{\text{steel}} &= \frac{k_{\text{steel}} A_{\text{steel}} \Delta T_{\text{steel}}}{L_{\text{steel}}} \\ &= \frac{(80 \text{ W/m} \cdot ^\circ\text{C})(7.85 \times 10^{-5} \text{ m}^2)(100^\circ\text{C} - T)}{0.25 \text{ m}} \\ &= (0.0251 \text{ W}/^\circ\text{C})(100^\circ\text{C} - T); \end{aligned}$$

$$\begin{aligned} P_{\text{Al}} &= \frac{k_{\text{Al}} A_{\text{Al}} \Delta T_{\text{Al}}}{L_{\text{Al}}} \\ &= \frac{(220 \text{ W/m} \cdot ^\circ\text{C})(7.85 \times 10^{-5} \text{ m}^2)(T - 20^\circ\text{C})}{0.25 \text{ m}} \\ &= (0.0691 \text{ W}/^\circ\text{C})(T - 20^\circ\text{C}). \end{aligned}$$

3. Set the two rates equal and solve for the unknown temperature:

$$\begin{aligned} (0.0691 \text{ W}/^\circ\text{C})(T - 20^\circ\text{C}) &= (0.0251 \text{ W}/^\circ\text{C})(100^\circ\text{C} - T) \\ T &= 41.3^\circ\text{C}. \end{aligned}$$

4. Calculate either rate:

$$P_{\text{steel}} = (0.0251 \text{ W}/^\circ\text{C})(100^\circ\text{C} - 41.3^\circ\text{C}) = 1.47 \text{ W}.$$

5. If desired, check your answer by calculating the other rate.

### Solution

- Recall that  $R = L/k$ . Now  $P = A\Delta T/R$ , or  $\Delta T = PR/A$ .
- We know that  $\Delta T_{\text{steel}} + \Delta T_{\text{Al}} = 100^\circ\text{C} - 20^\circ\text{C} = 80^\circ\text{C}$ . We also know that  $P_{\text{steel}} = P_{\text{Al}}$ , and we denote that rate of heat flow by  $P$ . Combine the equations:

$$\frac{PR_{\text{steel}}}{A} + \frac{PR_{\text{Al}}}{A} = 80^\circ\text{C}.$$

Thus, we can simply add  $R$  factors. Now,  $P = \frac{80^\circ\text{C}}{A(R_{\text{steel}} + R_{\text{Al}})}$ .

3. Find the  $R_s$  from the known quantities:

$$R_{\text{steel}} = 3.13 \times 10^{-3} \text{ m}^2 \cdot ^\circ\text{C}/\text{W}$$

and

$$R_{\text{Al}} = 1.14 \times 10^{-3} \text{ m}^2 \cdot ^\circ\text{C}/\text{W}.$$

- Substitute these values in to find  $P = 1.47 \text{ W}$  as before.
- Determine  $\Delta T$  for the aluminum rod (or for the steel rod) and use it to find  $T$  at the joint.

$$\Delta T_{\text{Al}} = \frac{PR_{\text{Al}}}{A} = \frac{(1.47 \text{ W})(1.14 \times 10^{-3} \text{ m}^2 \cdot ^\circ\text{C}/\text{W})}{7.85 \times 10^{-5} \text{ m}^2} = 21.3^\circ\text{C},$$

so  $T = 20^\circ\text{C} + 21.3^\circ\text{C} = 41.3^\circ\text{C}$ , as in Solution 1.

6. If desired, check by determining  $\Delta T$  for the other rod.

### Significance

In practice, adding  $R$  values is common, as in calculating the  $R$  value of an insulated wall. In the analogous situation in electronics, the resistance corresponds to  $AR$  in this problem and is additive even when the areas are

unequal, as is common in electronics. Our equation for heat conduction can be used only when the areas are equal; otherwise, we would have a problem in three-dimensional heat flow, which is beyond our scope.



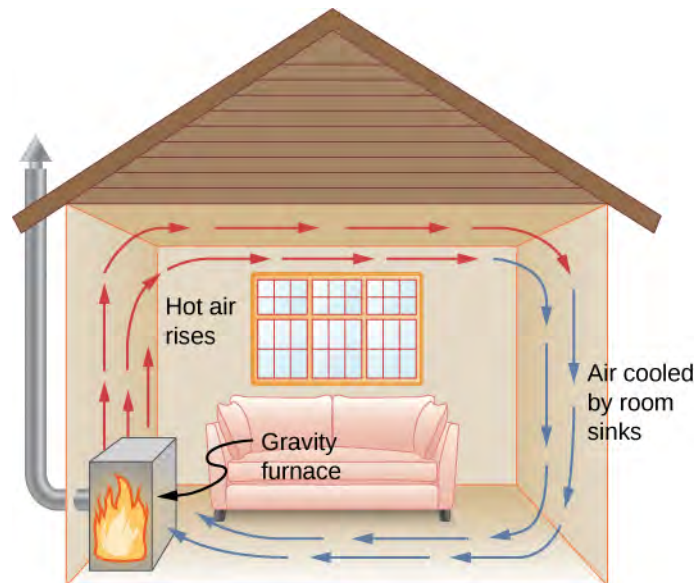
**1.7 Check Your Understanding** How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short times. For example, the temperature on Earth would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere were only through conduction. Also, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons. The next module discusses the important heat-transfer mechanism in such situations.

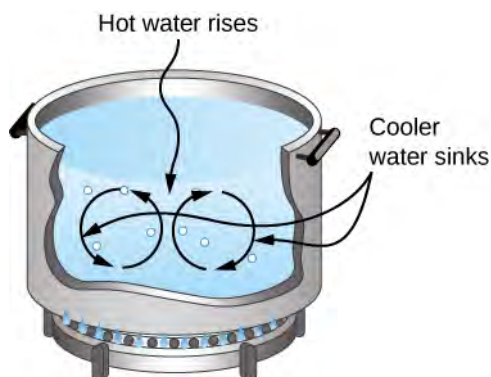
## Convection

In convection, thermal energy is carried by the large-scale flow of matter. It can be divided into two types. In *forced convection*, the flow is driven by fans, pumps, and the like. A simple example is a fan that blows air past you in hot surroundings and cools you by replacing the air heated by your body with cooler air. A more complicated example is the cooling system of a typical car, in which a pump moves coolant through the radiator and engine to cool the engine and a fan blows air to cool the radiator.

In *free or natural convection*, the flow is driven by buoyant forces: hot fluid rises and cold fluid sinks because density decreases as temperature increases. The house in **Figure 1.24** is kept warm by natural convection, as is the pot of water on the stove in **Figure 1.25**. Ocean currents and large-scale atmospheric circulation, which result from the buoyancy of warm air and water, transfer hot air from the tropics toward the poles and cold air from the poles toward the tropics. (Earth's rotation interacts with those flows, causing the observed eastward flow of air in the temperate zones.)



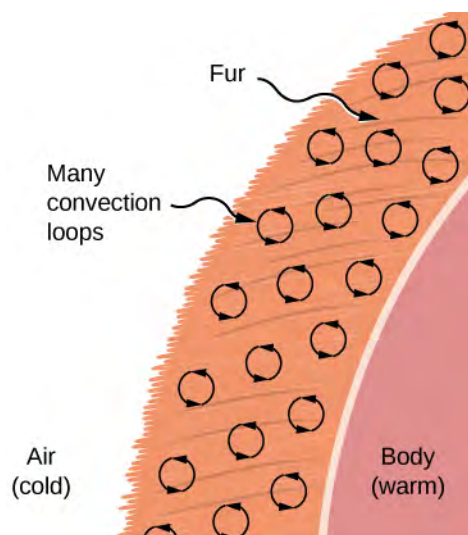
**Figure 1.24** Air heated by a so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can heat a home quite efficiently.



**Figure 1.25** Natural convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

 Natural convection like that of **Figure 1.24** and **Figure 1.25**, but acting on rock in Earth’s mantle, drives **plate tectonics** (<https://openstaxcollege.org/l/21platetecton>) that are the motions that have shaped Earth’s surface.

Convection is usually more complicated than conduction. Beyond noting that the convection rate is often approximately proportional to the temperature difference, we will not do any quantitative work comparable to the formula for conduction. However, we can describe convection qualitatively and relate convection rates to heat and time. However, air is a poor conductor. Therefore, convection dominates heat transfer by air, and the amount of available space for airflow determines whether air transfers heat rapidly or slowly. There is little heat transfer in a space filled with air with a small amount of other material that prevents flow. The space between the inside and outside walls of a typical American house, for example, is about 9 cm (3.5 in.)—large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. On the other hand, the gap between the two panes of a double-paned window is about 1 cm, which largely prevents convection and takes advantage of air’s low conductivity reduce heat loss. Fur, cloth, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection (**Figure 1.26**).



**Figure 1.26** Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen when convection is accompanied by a phase change. The combination allows us to

cool off by sweating even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

## Example 1.12

### Calculating the Flow of Mass during Convection

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (For simplicity, we assume this evaporation occurs when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

#### Strategy

Energy is needed for this phase change ( $Q = mL_v$ ). Thus, the energy loss per unit time is

$$\frac{Q}{t} = \frac{mL_v}{t} = 120 \text{ W} = 120 \text{ J/s}.$$

We divide both sides of the equation by  $L_v$  to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120 \text{ J/s}}{L_v}.$$

#### Solution

Insert the value of the latent heat from **Table 1.4**,  $L_v = 2430 \text{ kJ/kg} = 2430 \text{ J/g}$ . This yields

$$\frac{m}{t} = \frac{120 \text{ J/s}}{2430 \text{ J/g}} = 0.0494 \text{ g/s} = 2.96 \text{ g/min}.$$

#### Significance

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz.) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, possibly far from the ocean, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere (**Figure 1.27**). Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise to colder altitudes. More condensation occurs in these regions, which in turn drives the cloud even higher. This mechanism is an example of positive feedback, since the process reinforces and accelerates itself. It sometimes produces violent storms, with lightning and hail. The same mechanism drives hurricanes.



This **time-lapse video** (<https://openstaxcollege.org//21convthuncurr>) shows convection currents in a thunderstorm, including “rolling” motion similar to that of boiling water.



**Figure 1.27** Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: “Amada44”/Wikimedia Commons)



**1.8** **Check Your Understanding** Explain why using a fan in the summer feels refreshing.

## Radiation

You can feel the heat transfer from the Sun. The space between Earth and the Sun is largely empty, so the Sun warms us without any possibility of heat transfer by convection or conduction. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. In these examples, heat is transferred by radiation (**Figure 1.28**). That is, the hot body emits electromagnetic waves that are absorbed by the skin. No medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays.

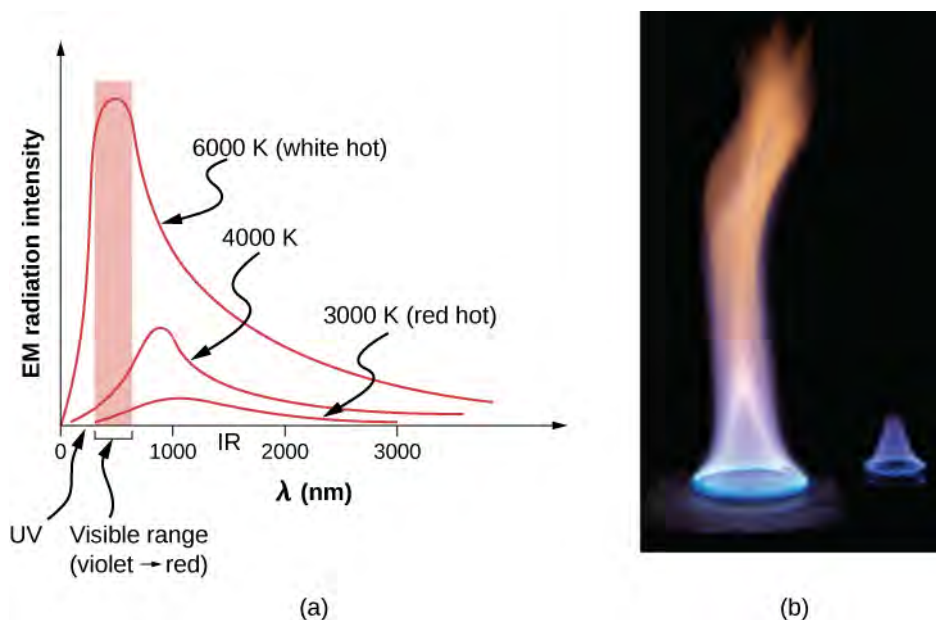


**Figure 1.28** Most of the heat transfer from this fire to the observers occurs through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so you can sense the presence of a fire without looking at it directly. (credit: Daniel O’Neil)

The energy of electromagnetic radiation varies over a wide range, depending on the wavelength: A shorter wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, higher temperatures produce more intensity at every wavelength but especially at shorter wavelengths. In visible light, wavelength determines color—red has the longest wavelength and violet the shortest—so a temperature change is accompanied by a color change. For example, an electric heating element on a stove glows from red to orange, while the higher-temperature steel in a

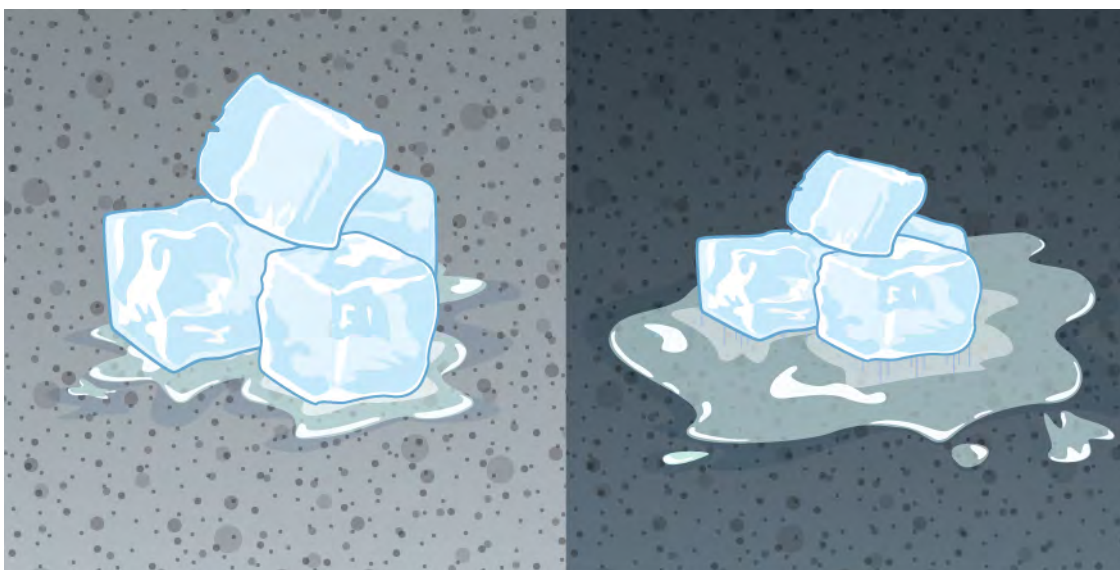


blast furnace glows from yellow to white. Infrared radiation is the predominant form radiated by objects cooler than the electric element and the steel. The radiated energy as a function of wavelength depends on its intensity, which is represented in **Figure 1.29** by the height of the distribution. (**Electromagnetic Waves** explains more about the electromagnetic spectrum, and **Photons and Matter Waves** (<http://cnx.org/content/m58757/latest/>) discusses why the decrease in wavelength corresponds to an increase in energy.)

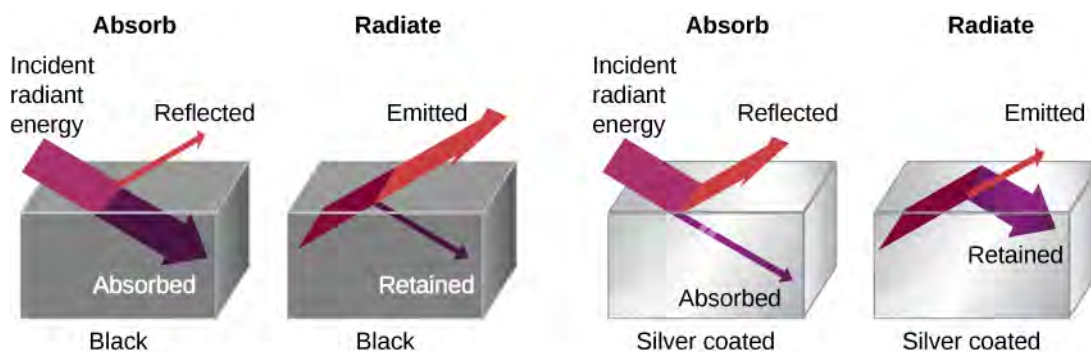


**Figure 1.29** (a) A graph of the spectrum of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts down in wavelength toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature.

The rate of heat transfer by radiation also depends on the object's color. Black is the most effective, and white is the least effective. On a clear summer day, black asphalt in a parking lot is hotter than adjacent gray sidewalk, because black absorbs better than gray (**Figure 1.30**). The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt is colder than the gray sidewalk, because black radiates the energy more rapidly than gray. A perfectly black object would be an *ideal radiator* and an *ideal absorber*, as it would capture all the radiation that falls on it. In contrast, a perfectly white object or a perfect mirror would reflect all radiation, and a perfectly transparent object would transmit it all (**Figure 1.31**). Such objects would not emit any radiation. Mathematically, the color is represented by the **emissivity**  $e$ . A “blackbody” radiator would have an  $e = 1$ , whereas a perfect reflector or transmitter would have  $e = 0$ . For real examples, tungsten light bulb filaments have an  $e$  of about 0.5, and carbon black (a material used in printer toner) has an emissivity of about 0.95.



**Figure 1.30** The darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.



**Figure 1.31** A black object is a good absorber and a good radiator, whereas a white, clear, or silver object is a poor absorber and a poor radiator.

To see that, consider a silver object and a black object that can exchange heat by radiation and are in thermal equilibrium. We know from experience that they will stay in equilibrium (the result of a principle that will be discussed at length in **Second Law of Thermodynamics**). For the black object's temperature to stay constant, it must emit as much radiation as it absorbs, so it must be as good at radiating as absorbing. Similar considerations show that the silver object must radiate as little as it absorbs. Thus, one property, emissivity, controls both radiation and absorption.

Finally, the radiated heat is proportional to the object's surface area, since every part of the surface radiates. If you knock apart the coals of a fire, the radiation increases noticeably due to an increase in radiating surface area.

The rate of heat transfer by emitted radiation is described by the **Stefan-Boltzmann law of radiation**:

$$P = \sigma A e T^4,$$

where  $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant, a combination of fundamental constants of nature;  $A$  is the surface area of the object; and  $T$  is its temperature in kelvins.

The proportionality to the *fourth power* of the absolute temperature is a remarkably strong temperature dependence. It allows the detection of even small temperature variations. Images called *thermographs* can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes (**Figure 1.32**), optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map Earth's temperature profile.



**Figure 1.32** A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: US Army)

The Stefan-Boltzmann equation needs only slight refinement to deal with a simple case of an object's absorption of radiation from its surroundings. Assuming that an object with a temperature  $T_1$  is surrounded by an environment with uniform temperature  $T_2$ , the **net rate of heat transfer by radiation** is

$$P_{\text{net}} = \sigma e A (T_2^4 - T_1^4), \quad (1.10)$$

where  $e$  is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black: The balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When  $T_2 > T_1$ , the quantity  $P_{\text{net}}$  is positive, that is, the net heat transfer is from hot to cold.

Before doing an example, we have a complication to discuss: different emissivities at different wavelengths. If the fraction of incident radiation an object reflects is the same at all visible wavelengths, the object is gray; if the fraction depends on the wavelength, the object has some other color. For instance, a red or reddish object reflects red light more strongly than other visible wavelengths. Because it absorbs less red, it radiates less red when hot. Differential reflection and absorption of wavelengths outside the visible range have no effect on what we see, but they may have physically important effects. Skin is a very good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared spectrum. Thus, in spite of the obvious variations in skin color, we are all nearly black in the infrared. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the effectiveness of night-vision scopes used by law enforcement and the military to detect human beings.

### Example 1.13

#### Calculating the Net Heat Transfer of a Person

What is the rate of heat transfer by radiation of an unclothed person standing in a dark room whose ambient temperature is  $22.0^\circ\text{C}$ ? The person has a normal skin temperature of  $33.0^\circ\text{C}$  and a surface area of  $1.50\text{ m}^2$ . The emissivity of skin is 0.97 in the infrared, the part of the spectrum where the radiation takes place.

#### Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

### Solution

Insert the temperature values  $T_2 = 295 \text{ K}$  and  $T_1 = 306 \text{ K}$ , so that

$$\begin{aligned}\frac{Q}{t} &= \sigma e A (T_2^4 - T_1^4) \\ &= (5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4)(0.97)(1.50 \text{ m}^2)[(295 \text{ K})^4 - (306 \text{ K})^4] \\ &= -99 \text{ J/s} = -99 \text{ W}.\end{aligned}$$

### Significance

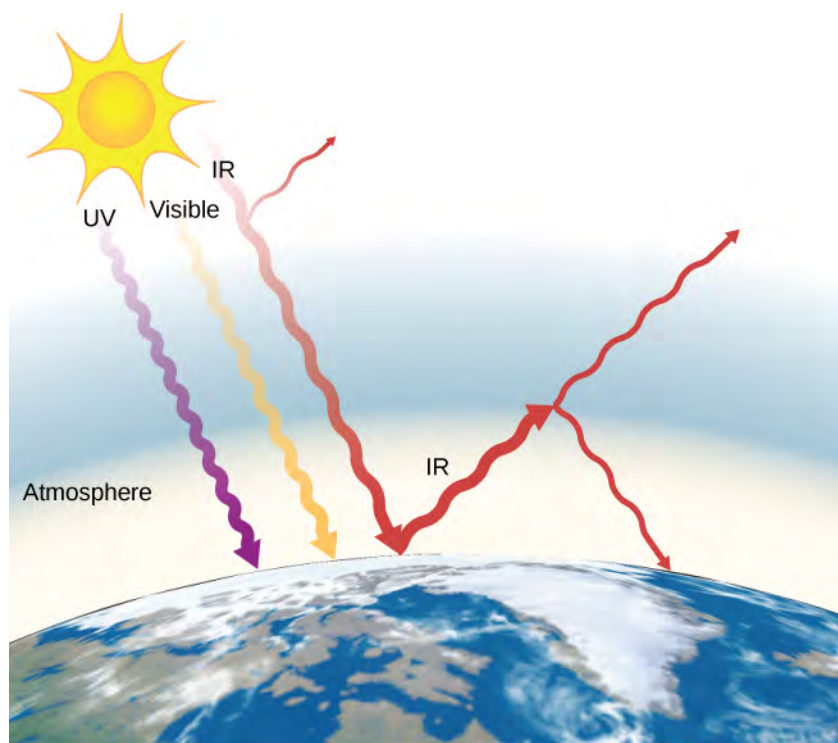
This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection are also transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by all mechanisms, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is light-colored) than skin.

The average temperature of Earth is the subject of much current discussion. Earth is in radiative contact with both the Sun and dark space, so we cannot use the equation for an environment at a uniform temperature. Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Conversely, dark space is very cold, about 3 K, so that Earth radiates energy into the dark sky. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

The average temperature of Earth is determined by its energy balance. To a first approximation, it is the temperature at which Earth radiates heat to space as fast as it receives energy from the Sun.


An important parameter in calculating the temperature of Earth is its emissivity ( $e$ ). On average, it is about 0.65, but calculation of this value is complicated by the great day-to-day variation in the highly reflective cloud coverage. Because clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. There is negative feedback (in which a change produces an effect that opposes that change) between clouds and heat transfer; higher temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature.

The often-mentioned **greenhouse effect** is directly related to the variation of Earth's emissivity with wavelength (**Figure 1.33**). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth and for making Venus unsuitable for human life. Most of the infrared radiation emitted from Earth is absorbed by carbon dioxide ( $\text{CO}_2$ ) and water ( $\text{H}_2\text{O}$ ) in the atmosphere and then re-radiated into outer space or back to Earth. Re-radiation back to Earth maintains its surface temperature about  $40^\circ\text{C}$  higher than it would be if there were no atmosphere. (The glass walls and roof of a greenhouse increase the temperature inside by blocking convective heat losses, not radiative losses.)



**Figure 1.33** The greenhouse effect is the name given to the increase of Earth's temperature due to absorption of radiation in the atmosphere. The atmosphere is transparent to incoming visible radiation and most of the Sun's infrared. The Earth absorbs that energy and re-emits it. Since Earth's temperature is much lower than the Sun's, it re-emits the energy at much longer wavelengths, in the infrared. The atmosphere absorbs much of that infrared radiation and radiates about half of the energy back down, keeping Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases such as carbon dioxide, and an increase in the concentration of these gases increases Earth's surface temperature.

The greenhouse effect is central to the discussion of global warming due to emission of carbon dioxide and methane (and other greenhouse gases) into Earth's atmosphere from industry, transportation, and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

 You can explore **a simulation of the greenhouse effect** (<https://openstaxcollege.org//21simgreneff>) that takes the point of view that the atmosphere scatters (redirects) infrared radiation rather than absorbing it and reradiating it. You may want to run the simulation first with no greenhouse gases in the atmosphere and then look at how adding greenhouse gases affects the infrared radiation from the Earth and the Earth's temperature.

### Problem-Solving Strategy: Effects of Heat Transfer

1. Examine the situation to determine what type of heat transfer is involved.
2. Identify the type(s) of heat transfer—conduction, convection, or radiation.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
4. Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).
5. Solve the appropriate equation for the quantity to be determined (the unknown).
6. For conduction, use the equation  $P = \frac{kA\Delta T}{d}$ . **Table 1.5** lists thermal conductivities. For convection, determine the amount of matter moved and the equation  $Q = mc\Delta T$ , along with  $Q = mL_f$  or  $Q = mL_V$  if a

substance changes phase. For radiation, the equation  $P_{\text{net}} = \sigma e A (T_2^4 - T_1^4)$  gives the net heat transfer rate.

7. Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.
8. Check the answer to see if it is reasonable. Does it make sense?



**1.9 Check Your Understanding** How much greater is the rate of heat radiation when a body is at the temperature  $40^\circ\text{C}$  than when it is at the temperature  $20^\circ\text{C}$ ?

## CHAPTER 1 REVIEW

### KEY TERMS

**absolute temperature scale** scale, such as Kelvin, with a zero point that is absolute zero

**absolute zero** temperature at which the average kinetic energy of molecules is zero

**calorie (cal)** energy needed to change the temperature of 1.00 g of water by 1.00 °C

**calorimeter** container that prevents heat transfer in or out

**calorimetry** study of heat transfer inside a container impervious to heat

**Celsius scale** temperature scale in which the freezing point of water is 0 °C and the boiling point of water is 100 °C

**coefficient of linear expansion** ( $\alpha$ ) material property that gives the change in length, per unit length, per 1-°C change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends to some degree on the temperature of the material

**coefficient of volume expansion** ( $\beta$ ) similar to  $\alpha$  but gives the change in volume, per unit volume, per 1-°C change in temperature

**conduction** heat transfer through stationary matter by physical contact

**convection** heat transfer by the macroscopic movement of fluid

**critical point** for a given substance, the combination of temperature and pressure above which the liquid and gas phases are indistinguishable

**critical pressure** pressure at the critical point

**critical temperature** temperature at the critical point

**degree Celsius** (°C) unit on the Celsius temperature scale

**degree Fahrenheit** (°F) unit on the Fahrenheit temperature scale

**emissivity** measure of how well an object radiates

**Fahrenheit scale** temperature scale in which the freezing point of water is 32 °F and the boiling point of water is 212 °F

**greenhouse effect** warming of the earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from Earth's surface and reradiate it in all directions, thus sending some of it back toward Earth

**heat** energy transferred solely due to a temperature difference

**heat of fusion** energy per unit mass required to change a substance from the solid phase to the liquid phase, or released when the substance changes from liquid to solid

**heat of sublimation** energy per unit mass required to change a substance from the solid phase to the vapor phase

**heat of vaporization** energy per unit mass required to change a substance from the liquid phase to the vapor phase

**heat transfer** movement of energy from one place or material to another as a result of a difference in temperature

**Kelvin scale (K)** temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

**kilocalorie (kcal)** energy needed to change the temperature of 1.00 kg of water between 14.5 °C and 15.5 °C

**latent heat coefficient** general term for the heats of fusion, vaporization, and sublimation

**mechanical equivalent of heat** work needed to produce the same effects as heat transfer

**net rate of heat transfer by radiation**  $P_{\text{net}} = \sigma e A (T_2^4 - T_1^4)$

**phase diagram** graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the phases of the substance occur

**radiation** energy transferred by electromagnetic waves directly as a result of a temperature difference

**rate of conductive heat transfer** rate of heat transfer from one material to another

**specific heat** amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C ; also called “specific heat capacity”

**Stefan-Boltzmann law of radiation**  $P = \sigma A e T^4$ , where  $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant,  $A$  is the surface area of the object,  $T$  is the absolute temperature, and  $e$  is the emissivity

**sublimation** phase change from solid to gas

**temperature** quantity measured by a thermometer, which reflects the mechanical energy of molecules in a system

**thermal conductivity** property of a material describing its ability to conduct heat

**thermal equilibrium** condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

**thermal expansion** change in size or volume of an object with change in temperature

**thermal stress** stress caused by thermal expansion or contraction

**triple point** pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

**vapor** gas at a temperature below the boiling temperature

**vapor pressure** pressure at which a gas coexists with its solid or liquid phase

**zeroth law of thermodynamics** law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

## KEY EQUATIONS

Linear thermal expansion

$$\Delta L = \alpha L \Delta T$$

Thermal expansion in two dimensions

$$\Delta A = 2\alpha A \Delta T$$

Thermal expansion in three dimensions

$$\Delta V = \beta V \Delta T$$

Heat transfer

$$Q = mc \Delta T$$

Transfer of heat in a calorimeter

$$Q_{\text{cold}} + Q_{\text{hot}} = 0$$

Heat due to phase change (melting and freezing)

$$Q = mL_f$$

Heat due to phase change (evaporation and condensation)

$$Q = mL_v$$

Rate of conductive heat transfer

$$P = \frac{kA(T_h - T_c)}{d}$$

Net rate of heat transfer by radiation

$$P_{\text{net}} = \sigma e A (T_2^4 - T_1^4)$$

## SUMMARY

### 1.1 Temperature and Thermal Equilibrium

- Temperature is operationally defined as the quantity measured by a thermometer. It is proportional to the average kinetic energy of atoms and molecules in a system.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy. Systems are in thermal equilibrium when they have the same temperature.
- The zeroth law of thermodynamics states that when two systems,  $A$  and  $B$ , are in thermal equilibrium with each other, and  $B$  is in thermal equilibrium with a third system  $C$ , then  $A$  is also in thermal equilibrium with  $C$ .



## 1.2 Thermometers and Temperature Scales

- Three types of thermometers are alcohol, liquid crystal, and infrared radiation (pyrometer).
- The three main temperature scales are Celsius, Fahrenheit, and Kelvin. Temperatures can be converted from one scale to another using temperature conversion equations.
- The three phases of water (ice, liquid water, and water vapor) can coexist at a single pressure and temperature known as the triple point.

## 1.3 Thermal Expansion

- Thermal expansion is the increase of the size (length, area, or volume) of a body due to a change in temperature, usually a rise. Thermal contraction is the decrease in size due to a change in temperature, usually a fall in temperature.
- Thermal stress is created when thermal expansion or contraction is constrained.

## 1.4 Heat Transfer, Specific Heat, and Calorimetry

- Heat and work are the two distinct methods of energy transfer.
- Heat transfer to an object when its temperature changes is often approximated well by  $Q = mc\Delta T$ , where  $m$  is the object's mass and  $c$  is the specific heat of the substance.

## 1.5 Phase Changes

- Most substances have three distinct phases (under ordinary conditions on Earth), and they depend on temperature and pressure.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures.
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling, freezing (or melting), and sublimation points.

## 1.6 Mechanisms of Heat Transfer

- Heat is transferred by three different methods: conduction, convection, and radiation.
- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer  $P$  (energy per unit time) is proportional to the temperature difference  $T_h - T_c$  and the contact area  $A$  and inversely proportional to the distance  $d$  between the objects.
- Convection is heat transfer by the macroscopic movement of mass. Convection can be natural or forced, and generally transfers thermal energy faster than conduction. Convection that occurs along with a phase change can transfer energy from cold regions to warm ones.
- Radiation is heat transfer through the emission or absorption of electromagnetic waves.
- The rate of radiative heat transfer is proportional to the emissivity  $e$ . For a perfect blackbody,  $e = 1$ , whereas a perfectly white, clear, or reflective body has  $e = 0$ , with real objects having values of  $e$  between 1 and 0.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

$$P = \sigma eAT^4,$$

where  $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$  is the Stefan-Boltzmann constant and  $e$  is the emissivity of the body. The net rate of heat transfer from an object by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma eA(T_2^4 - T_1^4),$$

where  $T_1$  is the temperature of the object surrounded by an environment with uniform temperature  $T_2$  and  $e$  is the

emissivity of the object.

## CONCEPTUAL QUESTIONS

### 1.1 Temperature and Thermal Equilibrium

1. What does it mean to say that two systems are in thermal equilibrium?
2. Give an example in which  $A$  has some kind of non-thermal equilibrium relationship with  $B$ , and  $B$  has the same relationship with  $C$ , but  $A$  does not have that relationship with  $C$ .

### 1.2 Thermometers and Temperature Scales

3. If a thermometer is allowed to come to equilibrium with the air, and a glass of water is not in equilibrium with the air, what will happen to the thermometer reading when it is placed in the water?
4. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

### 1.3 Thermal Expansion

5. Pouring cold water into hot glass or ceramic cookware can easily break it. What causes the breaking? Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.
6. One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.
7. Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.
8. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.
9. Calculate the length of a 1-meter rod of a material with thermal expansion coefficient  $\alpha$  when the temperature is raised from 300 K to 600 K. Taking your answer as the new initial length, find the length after the rod is cooled back down to 300 K. Is your answer 1 meter? Should it be? How can you account for the result you got?
10. Noting the large stresses that can be caused by thermal expansion, an amateur weapon inventor decides to use it to

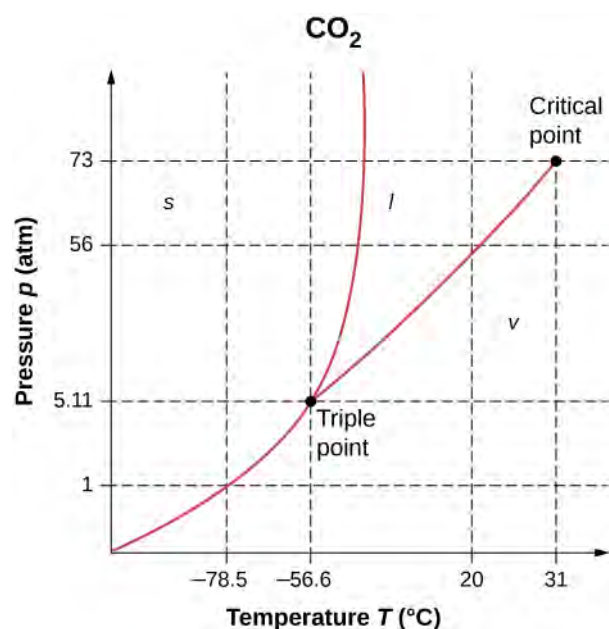
make a new kind of gun. He plans to jam a bullet against an aluminum rod inside a closed invar tube. When he heats the tube, the rod will expand more than the tube and a very strong force will build up. Then, by a method yet to be determined, he will open the tube in a split second and let the force of the rod launch the bullet at very high speed. What is he overlooking?

### 1.4 Heat Transfer, Specific Heat, and Calorimetry

11. How is heat transfer related to temperature?
12. Describe a situation in which heat transfer occurs.
13. When heat transfers into a system, is the energy stored as heat? Explain briefly.
14. The brakes in a car increase in temperature by  $\Delta T$  when bringing the car to rest from a speed  $v$ . How much greater would  $\Delta T$  be if the car initially had twice the speed? You may assume the car stops fast enough that no heat transfers out of the brakes.

### 1.5 Phase Changes

15. A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?
16. As shown below, which is the phase diagram for carbon dioxide, what is the vapor pressure of solid carbon dioxide (dry ice) at  $-78.5^\circ\text{C}$ ? (Note that the axes in the figure are nonlinear and the graph is not to scale.)



17. Can carbon dioxide be liquefied at room temperature (20 °C)? If so, how? If not, why not? (See the phase diagram in the preceding problem.)

18. What is the distinction between gas and vapor?

19. Heat transfer can cause temperature and phase changes. What else can cause these changes?

20. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0 °C, in the vicinity of large bodies of water?

21. What is the temperature of ice right after it is formed by freezing water?

22. If you place 0 °C ice into 0 °C water in an insulated container, what will the net result be? Will there be less ice and more liquid water, or more ice and less liquid water, or will the amounts stay the same?

23. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

24. In Miami, Florida, which has a very humid climate and numerous bodies of water nearby, it is unusual for temperatures to rise above about 38 °C (100 °F). In the desert climate of Phoenix, Arizona, however, temperatures rise above that almost every day in July and August. Explain how the evaporation of water helps limit high temperatures in humid climates.

25. In winter, it is often warmer in San Francisco than in Sacramento, 150 km inland. In summer, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

26. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

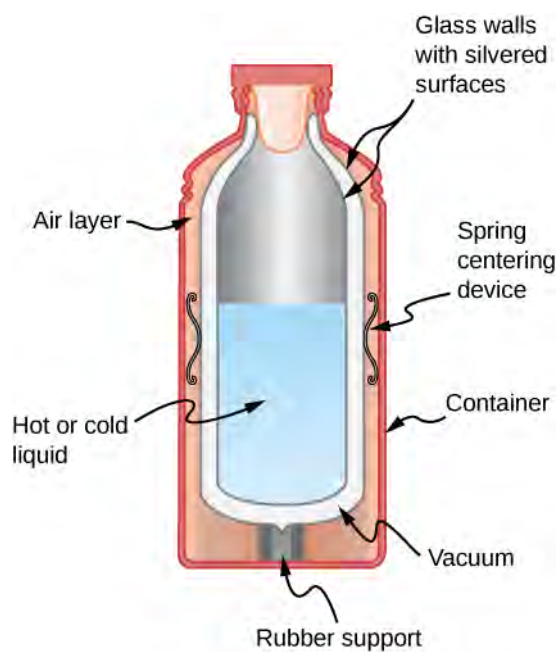
27. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublime, with some crystals lingering for a while and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from **Table 1.4**.

## 1.6 Mechanisms of Heat Transfer

28. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

29. When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will those processes have on a person in a 40.0-°C hot tub?

30. Shown below is a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.



**31.** Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while the surface only a few centimeters away is safe to touch. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

**32.** Loose-fitting white clothing covering most of the body, shown below, is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.



**33.** One way to make a fireplace more energy-efficient is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved.

**34.** On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

**35.** When watching a circus during the day in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.

**36.** Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and the Moon and are cooled to very low temperatures. Why must the sensors be at low temperature?

**37.** Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine? What does it measure if it is not?

**38.** Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

**39.** Your house will be empty for a while in cold weather, and you want to save energy and money. Should you turn the thermostat down to the lowest level that will protect the house from damage such as freezing pipes, or leave it at the normal temperature? (If you don't like coming back to a cold house, imagine that a timer controls the heating system so the house will be warm when you get back.) Explain your answer.

**40.** You pour coffee into an unlidded cup, intending to drink it 5 minutes later. You can add cream when you pour the cup or right before you drink it. (The cream is at the same temperature either way. Assume that the cream and coffee come into thermal equilibrium with each other very quickly.) Which way will give you hotter coffee? What feature of this question is different from the previous one?

**41.** Broiling is a method of cooking by radiation, which produces somewhat different results from cooking by conduction or convection. A gas flame or electric heating element produces a very high temperature close to the food and *above* it. Why is radiation the dominant heat-transfer method in this situation?

**42.** On a cold winter morning, why does the metal of a bike feel colder than the wood of a porch?

## PROBLEMS

### 1.2 Thermometers and Temperature Scales

43. While traveling outside the United States, you feel sick. A companion gets you a thermometer, which says your temperature is 39. What scale is that on? What is your Fahrenheit temperature? Should you seek medical help?
44. What are the following temperatures on the Kelvin scale?
- 68.0 °F, an indoor temperature sometimes recommended for energy conservation in winter
  - 134 °F, one of the highest atmospheric temperatures ever recorded on Earth (Death Valley, California, 1913)
  - 9890 °F, the temperature of the surface of the Sun
45. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when it decreases by 40.0 °F? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees
46. An Associated Press article on climate change said, “Some of the ice shelf’s disappearance was probably during times when the planet was 36 degrees Fahrenheit (2 degrees Celsius) to 37 degrees Fahrenheit (3 degrees Celsius) warmer than it is today.” What mistake did the reporter make?
47. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?
48. A person taking a reading of the temperature in a freezer in Celsius makes two mistakes: first omitting the negative sign and then thinking the temperature is Fahrenheit. That is, the person reads  $-x$  °C as  $x$  °F. Oddly enough, the result is the correct Fahrenheit temperature. What is the original Celsius reading? Round your answer to three significant figures.
50. How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by 15 °C? Its original height is 321 m and you can assume it is made of steel.
51. What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from 37.0 °C to 40.0 °C, assuming the mercury is constrained to a cylinder but unconstrained in length? Your answer will show why thermometers contain bulbs at the bottom instead of simple columns of liquid.
52. How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature 35.0 °C greater than when they were laid? Their original length is 10.0 m.
53. You are looking to buy a small piece of land in Hong Kong. The price is “only” \$60,000 per square meter. The land title says the dimensions are 20 m × 30 m. By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature was 20 °C above the temperature that the tape measure was designed for? The dimensions of the land do not change.
54. Global warming will produce rising sea levels partly due to melting ice caps and partly due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of 1.00 °C. Assume the column is not free to expand sideways. As a model of the ocean, that is a reasonable approximation, as only parts of the ocean very close to the surface can expand sideways onto land, and only to a limited degree. As another approximation, neglect the fact that ocean warming is not uniform with depth.
55. (a) Suppose a meter stick made of steel and one made of aluminum are the same length at 0 °C. What is their difference in length at 22.0 °C? (b) Repeat the calculation for two 30.0-m-long surveyor’s tapes.

### 1.3 Thermal Expansion

49. The height of the Washington Monument is measured to be 170.00 m on a day when the temperature is 35.0 °C. What will its height be on a day when the temperature falls to  $-10.0$  °C? Although the monument is made of limestone, assume that its coefficient of thermal expansion is the same as that of marble. Give your answer to five significant figures.
56. (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of 5.00 °C, how much will overflow when the alcohol’s temperature reaches the room temperature of 22.0 °C? (b) How much less water would overflow under the same conditions?
57. Most cars have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at 10.0 °C. What volume of radiator fluid will overflow

when the radiator and fluid reach a temperature of  $95.0\text{ }^{\circ}\text{C}$ , given that the fluid's volume coefficient of expansion is  $\beta = 400 \times 10^{-6}/^{\circ}\text{C}$ ? (Your answer will be a conservative estimate, as most car radiators have operating temperatures greater than  $95.0\text{ }^{\circ}\text{C}$ ).

**58.** A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the  $350\text{ cm}^3$  of coffee is in a 7.00-cm-diameter cup and decreases in temperature from  $95.0\text{ }^{\circ}\text{C}$  to  $45.0\text{ }^{\circ}\text{C}$ . (Most of the drop in level is actually due to escaping bubbles of air.)

**59.** The density of water at  $0\text{ }^{\circ}\text{C}$  is very nearly  $1000\text{ kg/m}^3$  (it is actually  $999.84\text{ kg/m}^3$ ), whereas the density of ice at  $0\text{ }^{\circ}\text{C}$  is  $917\text{ kg/m}^3$ . Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.)

**60.** Show that  $\beta = 3\alpha$ , by calculating the infinitesimal change in volume  $dV$  of a cube with sides of length  $L$  when the temperature changes by  $dT$ .

#### 1.4 Heat Transfer, Specific Heat, and Calorimetry

**61.** On a hot day, the temperature of an 80,000-L swimming pool increases by  $1.50\text{ }^{\circ}\text{C}$ . What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

**62.** To sterilize a 50.0-g glass baby bottle, we must raise its temperature from  $22.0\text{ }^{\circ}\text{C}$  to  $95.0\text{ }^{\circ}\text{C}$ . How much heat transfer is required?

**63.** The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at  $20.0\text{ }^{\circ}\text{C}$ : (a) water; (b) concrete; (c) steel; and (d) mercury.

**64.** Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her

hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

**65.** A 0.250-kg block of a pure material is heated from  $20.0\text{ }^{\circ}\text{C}$  to  $65.0\text{ }^{\circ}\text{C}$  by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

**66.** Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

**67.** (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a  $54.9\text{ }^{\circ}\text{C}$  temperature increase? Assume the process takes place in an ideal calorimeter, in other words a perfectly insulated container. (b) Compare your answer to the following labeling information found on a package of dry roasted peanuts: a serving of 33 g contains 200 calories. Comment on whether the values are consistent.

**68.** Following vigorous exercise, the body temperature of an 80.0 kg person is  $40.0\text{ }^{\circ}\text{C}$ . At what rate in watts must the person transfer thermal energy to reduce the body temperature to  $37.0\text{ }^{\circ}\text{C}$  in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? (1 watt = 1 joule/second or  $1\text{ W} = 1\text{ J/s}$ )

**69.** In a study of healthy young men<sup>[1]</sup>, doing 20 push-ups in 1 minute burned an amount of energy per kg that for a 70.0-kg man corresponds to 8.06 calories (kcal). How much would a 70.0-kg man's temperature rise if he did not lose any heat during that time?

**70.** A 1.28-kg sample of water at  $10.0\text{ }^{\circ}\text{C}$  is in a calorimeter. You drop a piece of steel with a mass of 0.385 kg at  $215\text{ }^{\circ}\text{C}$  into it. After the sizzling subsides, what is the final equilibrium temperature? (Make the reasonable assumptions that any steam produced condenses into liquid water during the process of equilibration and that the evaporation and condensation don't affect the outcome, as we'll see in the next section.)

**71.** Repeat the preceding problem, assuming the water

1. JW Vezina, "An examination of the differences between two methods of estimating energy expenditure in resistance training activities," *Journal of Strength and Conditioning Research*, April 28, 2014, <http://www.ncbi.nlm.nih.gov/pubmed/24402448>

is in a glass beaker with a mass of 0.200 kg, which in turn is in a calorimeter. The beaker is initially at the same temperature as the water. Before doing the problem, should the answer be higher or lower than the preceding answer? Comparing the mass and specific heat of the beaker to those of the water, do you think the beaker will make much difference?

### 1.5 Phase Changes

72. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at  $0^{\circ}\text{C}$  if their heat of fusion is the same as that of water?

73. A bag containing  $0^{\circ}\text{C}$  ice is much more effective in absorbing energy than one containing the same amount of  $0^{\circ}\text{C}$  water. (a) How much heat transfer is necessary to raise the temperature of 0.800 kg of water from  $0^{\circ}\text{C}$  to  $30.0^{\circ}\text{C}$ ? (b) How much heat transfer is required to first melt 0.800 kg of  $0^{\circ}\text{C}$  ice and then raise its temperature? (c) Explain how your answer supports the contention that the ice is more effective.

74. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from  $30.0^{\circ}\text{C}$  to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W?

75. Condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of vapor condense on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs. Use  $L_v$  for water at  $37^{\circ}\text{C}$  as a better approximation than  $L_v$  for water at  $100^{\circ}\text{C}$ .

76. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at  $0^{\circ}\text{C}$  and completely melts to  $0^{\circ}\text{C}$  water in exactly one day?

77. On a certain dry sunny day, a swimming pool's temperature would rise by  $1.50^{\circ}\text{C}$  if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

78. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from  $-20.0^{\circ}\text{C}$  to  $130.0^{\circ}\text{C}$ , including the energy needed for phase changes? (b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer? (c) Make a graph of temperature versus time for this process.

79. In 1986, an enormous iceberg broke away from the

Ross Ice Shelf in Antarctica. It was an approximately rectangular prism 160 km long, 40.0 km wide, and 250 m thick. (a) What is the mass of this iceberg, given that the density of ice is  $917\text{ kg/m}^3$ ? (b) How much heat transfer (in joules) is needed to melt it? (c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of  $100\text{ W/m}^2$ , 12.00 h per day?

80. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee and the cup from  $95.0^{\circ}\text{C}$  to  $45.0^{\circ}\text{C}$ ? Assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 kcal/g). Neglect heat losses through processes other than evaporation, as well as the change in mass of the coffee as it cools. Do the latter two assumptions cause your answer to be higher or lower than the true answer?

81. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases  $2.80 \times 10^7\text{ J}$  of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water's temperature rises from  $20.0^{\circ}\text{C}$  to  $100^{\circ}\text{C}$ , it boils, and the resulting steam's temperature rises to  $300^{\circ}\text{C}$  at constant pressure. (b) Discuss additional complications caused by the fact that crude oil is less dense than water.

82. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

83. To help prevent frost damage, 4.00 kg of water at  $0^{\circ}\text{C}$  is sprayed onto a fruit tree. (a) How much heat transfer occurs as the water freezes? (b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be  $3.35\text{ kJ/kg}\cdot^{\circ}\text{C}$ , and assume that no phase change occurs in the tree.

84. A 0.250-kg aluminum bowl holding 0.800 kg of soup at  $25.0^{\circ}\text{C}$  is placed in a freezer. What is the final temperature if 388 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water?

85. A 0.0500-kg ice cube at  $-30.0^{\circ}\text{C}$  is placed in 0.400 kg of  $35.0^{\circ}\text{C}$  water in a very well-insulated container. What is the final temperature?

**86.** If you pour 0.0100 kg of  $20.0^{\circ}\text{C}$  water onto a 1.20-kg block of ice (which is initially at  $-15.0^{\circ}\text{C}$ ), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

**87.** Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of  $500^{\circ}\text{C}$  granite must be placed in 4.00 kg of  $15.0^{\circ}\text{C}$  water to bring its temperature to  $100^{\circ}\text{C}$ , if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings.

**88.** What would the final temperature of the pan and water be in **Example 1.7** if 0.260 kg of water were placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?

### 1.6 Mechanisms of Heat Transfer

**89.** (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The walls' surface area is  $120\text{ m}^2$  and their inside surface is at  $18.0^{\circ}\text{C}$ , while their outside surface is at  $5.00^{\circ}\text{C}$ . (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?

**90.** The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a  $3.00\text{-m}^2$  window that is 0.634 cm thick (1/4 in.) if the temperatures of the inner and outer surfaces are  $5.00^{\circ}\text{C}$  and  $-10.0^{\circ}\text{C}$ , respectively. (This rapid rate will not be maintained—the inner surface will cool, even to the point of frost formation.)

**91.** Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is  $37.0^{\circ}\text{C}$ , the skin temperature is  $34.0^{\circ}\text{C}$ , the thickness of the fatty tissues between the core and the skin averages 1.00 cm, and the surface area is  $1.40\text{ m}^2$ .

**92.** Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of  $80.0\text{ cm}^2$  with each foot. Both the ceramic and the carpet are 2.00 cm thick and are  $10.0^{\circ}\text{C}$  on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at  $33.0^{\circ}\text{C}$ ?

**93.** A man consumes 3000 kcal of food in one day, converting most of it to thermal energy to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

**94.** A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is  $300\text{ kg/m}^3$ . The area of contact is  $25.0\text{ cm}^2$ , the temperature of the coals is  $700^{\circ}\text{C}$ , and the time in contact is 1.00 s. Ignore the evaporative cooling of sweat.

**95.** (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a  $1.40\text{-m}^2$  surface area? Assume that the animal's skin temperature is  $32.0^{\circ}\text{C}$ , that the air temperature is  $-5.00^{\circ}\text{C}$ , and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?

**96.** A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in  $-1.00^{\circ}\text{C}$  water. The walrus's internal core temperature is  $37.0^{\circ}\text{C}$ , and it has a surface area of  $2.00\text{ m}^2$ . What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?

**97.** Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of  $10.0\text{ m}^2$  and a thermal conductivity twice that of glass wool with the rate of heat conduction through a 0.750-cm-thick window that has an area of  $2.00\text{ m}^2$ , assuming the same temperature difference across each.

**98.** Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is  $1.40\text{ m}^2$ ?

**99.** Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in **Example 1.11**, what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.

**100.** One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose a single-story cubical house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If



you added an extra 8.0 cm of fiberglass to the attic, by what percentage would the heating cost of the house drop? Take the house to have dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors, and assume that the interior is uniformly at one temperature and the exterior is uniformly at another.

**101.** Many decisions are made on the basis of the payback period: the time it will take through savings to equal the

capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as 2 years.) Suppose you wish to install the extra insulation in the preceding problem. If energy cost \$1.00 per million joules and the insulation was \$4.00 per square meter, then calculate the simple payback time. Take the average  $\Delta T$  for the 120-day heating season to be 15.0 °C.

## ADDITIONAL PROBLEMS

**102.** In 1701, the Danish astronomer Ole Rømer proposed a temperature scale with two fixed points, freezing water at 7.5 degrees, and boiling water at 60.0 degrees. What is the boiling point of oxygen, 90.2 K, on the Rømer scale?

**103.** What is the percent error of thinking the melting point of tungsten is 3695 °C instead of the correct value of 3695 K?

**104.** An engineer wants to design a structure in which the difference in length between a steel beam and an aluminum beam remains at 0.500 m regardless of temperature, for ordinary temperatures. What must the lengths of the beams be?

**105.** How much stress is created in a steel beam if its temperature changes from  $-15\text{ °C}$  to  $40\text{ °C}$  but it cannot expand? For steel, the Young's modulus  $Y = 210 \times 10^9\text{ N/m}^2$  from **Stress, Strain, and Elastic Modulus** (<http://cnx.org/content/m58342/latest/#fs-id1163713086230>). (Ignore the change in area resulting from the expansion.)

**106.** A brass rod ( $Y = 90 \times 10^9\text{ N/m}^2$ ), with a diameter of 0.800 cm and a length of 1.20 m when the temperature is  $25\text{ °C}$ , is fixed at both ends. At what temperature is the force in it at 36,000 N?

**107.** A mercury thermometer still in use for meteorology has a bulb with a volume of  $0.780\text{ cm}^3$  and a tube for the mercury to expand into of inside diameter 0.130 mm. (a) Neglecting the thermal expansion of the glass, what is the spacing between marks  $1\text{ °C}$  apart? (b) If the thermometer is made of ordinary glass (not a good idea), what is the spacing?

**108.** Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails

(1 watt = 1 joule/second or  $1\text{ W} = 1\text{ J/s}$  and  $1\text{ MW} = 1\text{ megawatt}$ ). (a) Calculate the rate of temperature increase in degrees Celsius per second ( $\text{°C/s}$ ) if the mass of the reactor core is  $1.60 \times 10^5\text{ kg}$  and it has an average specific heat of  $0.3349\text{ kJ/kg} \cdot \text{°C}$ . (b) How long would it take to obtain a temperature increase of  $2000\text{ °C}$ , which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the 500,000-kg steel containment vessel would also begin to heat up.)

**109.** You leave a pastry in the refrigerator on a plate and ask your roommate to take it out before you get home so you can eat it at room temperature, the way you like it. Instead, your roommate plays video games for hours. When you return, you notice that the pastry is still cold, but the game console has become hot. Annoyed, and knowing that the pastry will not be good if it is microwaved, you warm up the pastry by unplugging the console and putting it in a clean trash bag (which acts as a perfect calorimeter) with the pastry on the plate. After a while, you find that the equilibrium temperature is a nice, warm  $38.3\text{ °C}$ . You know that the game console has a mass of 2.1 kg. Approximate it as having a uniform initial temperature of  $45\text{ °C}$ . The pastry has a mass of 0.16 kg and a specific heat of  $3.0\text{ kJ/(kg} \cdot \text{°C)}$ , and is at a uniform initial temperature of  $4.0\text{ °C}$ . The plate is at the same temperature and has a mass of 0.24 kg and a specific heat of  $0.90\text{ J/(kg} \cdot \text{°C)}$ . What is the specific heat of the console?

**110.** Two solid spheres, A and B, made of the same material, are at temperatures of  $0\text{ °C}$  and  $100\text{ °C}$ , respectively. The spheres are placed in thermal contact in an ideal calorimeter, and they reach an equilibrium temperature of  $20\text{ °C}$ . Which is the bigger sphere? What is the ratio of their diameters?

**111.** In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of  $808 \text{ kg/m}^3$ . (a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to  $3.00^\circ\text{C}$ . (Use  $c_p$  and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies. (b) What is this heat transfer rate in kilowatt-hours? (c) Compare the amount of cooling obtained from melting an identical mass of  $0^\circ\text{C}$  ice with that from evaporating the liquid nitrogen.

**112.** Some gun fanciers make their own bullets, which involves melting lead and casting it into lead slugs. How much heat transfer is needed to raise the temperature and melt  $0.500 \text{ kg}$  of lead, starting from  $25.0^\circ\text{C}$ ?

**113.** A  $0.800\text{-kg}$  iron cylinder at a temperature of  $1.00 \times 10^3^\circ\text{C}$  is dropped into an insulated chest of  $1.00 \text{ kg}$  of ice at its melting point. What is the final temperature, and how much ice has melted?

**114.** Repeat the preceding problem with  $2.00 \text{ kg}$  of ice instead of  $1.00 \text{ kg}$ .

**115.** Repeat the preceding problem with  $0.500 \text{ kg}$  of ice, assuming that the ice is initially in a copper container of mass  $1.50 \text{ kg}$  in equilibrium with the ice.

**116.** A  $30.0\text{-g}$  ice cube at its melting point is dropped into an aluminum calorimeter of mass  $100.0 \text{ g}$  in equilibrium at  $24.0^\circ\text{C}$  with  $300.0 \text{ g}$  of an unknown liquid. The final temperature is  $4.0^\circ\text{C}$ . What is the heat capacity of the liquid?

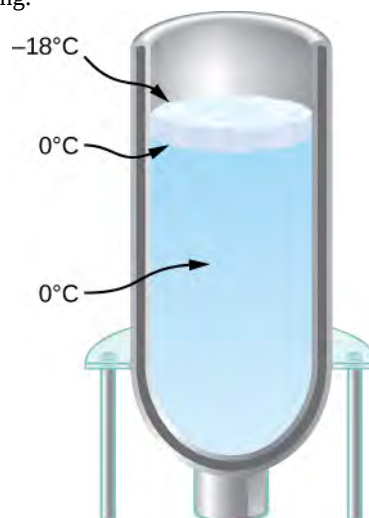
**117.** (a) Calculate the rate of heat conduction through a double-paned window that has a  $1.50\text{-m}^2$  area and is made of two panes of  $0.800\text{-cm}$ -thick glass separated by a  $1.00\text{-cm}$  air gap. The inside surface temperature is  $15.0^\circ\text{C}$ , while that on the outside is  $-10.0^\circ\text{C}$ . (*Hint:* There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.) (b) Calculate the rate of heat conduction through a  $1.60\text{-cm}$ -thick window of the same area and with the same temperatures. Compare your answer with that for part (a).

**118.** (a) An exterior wall of a house is  $3 \text{ m}$  tall and  $10 \text{ m}$  wide. It consists of a layer of drywall with an  $R$  factor of  $0.56$ , a layer  $3.5$  inches thick filled with fiberglass batts, and a layer of insulated siding with an  $R$  factor of  $2.6$ . The wall is built so well that there are no leaks of air through it. When the inside of the wall is at  $22^\circ\text{C}$  and the outside is at

$-2^\circ\text{C}$ , what is the rate of heat flow through the wall? (b) More realistically, the  $3.5\text{-inch}$  space also contains 2-by-4 studs—wooden boards  $1.5$  inches by  $3.5$  inches oriented so that  $3.5\text{-inch}$  dimension extends from the drywall to the siding. They are “on 16-inch centers,” that is, the centers of the studs are 16 inches apart. What is the heat current in this situation? Don’t worry about one stud more or less.

**119.** For the human body, what is the rate of heat transfer by conduction through the body’s tissue with the following conditions: the tissue thickness is  $3.00 \text{ cm}$ , the difference in temperature is  $2.00^\circ\text{C}$ , and the skin area is  $1.50 \text{ m}^2$ . How does this compare with the average heat transfer rate to the body resulting from an energy intake of about  $2400 \text{ kcal}$  per day? (No exercise is included.)

**120.** You have a Dewar flask (a laboratory vacuum flask) that has an open top and straight sides, as shown below. You fill it with water and put it into the freezer. It is effectively a perfect insulator, blocking all heat transfer, except on the top. After a time, ice forms on the surface of the water. The liquid water and the bottom surface of the ice, in contact with the liquid water, are at  $0^\circ\text{C}$ . The top surface of the ice is at the same temperature as the air in the freezer,  $-18^\circ\text{C}$ . Set the rate of heat flow through the ice equal to the rate of loss of heat of fusion as the water freezes. When the ice layer is  $0.700 \text{ cm}$  thick, find the rate in  $\text{m/s}$  at which the ice is thickening.



**121.** An infrared heater for a sauna has a surface area of  $0.050 \text{ m}^2$  and an emissivity of  $0.84$ . What temperature must it run at if the required power is  $360 \text{ W}$ ? Neglect the temperature of the environment.

**122.** (a) Determine the power of radiation from the Sun by noting that the intensity of the radiation at the distance of Earth is  $1370 \text{ W/m}^2$ . *Hint:* That intensity will be found everywhere on a spherical surface with radius equal to that of Earth’s orbit. (b) Assuming that the Sun’s temperature is

5780 K and that its emissivity is 1, find its radius.

## CHALLENGE PROBLEMS

**123.** A pendulum is made of a rod of length  $L$  and negligible mass, but capable of thermal expansion, and a weight of negligible size. (a) Show that when the temperature increases by  $dT$ , the period of the pendulum increases by a fraction  $\alpha LdT/2$ . (b) A clock controlled by a brass pendulum keeps time correctly at  $10^\circ\text{C}$ . If the room temperature is  $30^\circ\text{C}$ , does the clock run faster or slower? What is its error in seconds per day?

**124.** At temperatures of a few hundred kelvins the specific heat capacity of copper approximately follows the empirical formula  $c = \alpha + \beta T + \delta T^{-2}$ , where  $\alpha = 349 \text{ J/kg} \cdot \text{K}$ ,  $\beta = 0.107 \text{ J/kg} \cdot \text{K}^2$ , and  $\delta = 4.58 \times 10^5 \text{ J} \cdot \text{kg} \cdot \text{K}$ . How much heat is needed to raise the temperature of a 2.00-kg piece of copper from  $20^\circ\text{C}$  to  $250^\circ\text{C}$ ?

**125.** In a calorimeter of negligible heat capacity, 200 g of steam at  $150^\circ\text{C}$  and 100 g of ice at  $-40^\circ\text{C}$  are mixed. The pressure is maintained at 1 atm. What is the final temperature, and how much steam, ice, and water are present?

**126.** An astronaut performing an extra-vehicular activity (space walk) shaded from the Sun is wearing a spacesuit that can be approximated as perfectly white ( $e = 0$ ) except for a  $5 \text{ cm} \times 8 \text{ cm}$  patch in the form of the astronaut's national flag. The patch has emissivity 0.300. The spacesuit under the patch is 0.500 cm thick, with a thermal conductivity  $k = 0.0600 \text{ W/m} \cdot \text{K}$ , and its inner surface is at a temperature of  $20.0^\circ\text{C}$ . What is the temperature of the patch, and what is the rate of heat loss through it? Assume the patch is so thin that its outer surface is at the same temperature as the outer surface of the spacesuit under it. Also assume the temperature of outer space is 0 K. You will get an equation that is very hard to solve in closed form, so you can solve it numerically with a graphing calculator, with software, or even by trial and error with a calculator.

**127.** The goal in this problem is to find the growth of an ice layer as a function of time. Call the thickness of the ice layer  $L$ . (a) Derive an equation for  $dL/dt$  in terms of  $L$ , the temperature  $T$  above the ice, and the properties of ice (which you can leave in symbolic form instead of substituting the numbers). (b) Solve this differential equation assuming that at  $t = 0$ , you have  $L = 0$ . If you have studied differential equations, you will know a technique for solving equations of this type: manipulate the

equation to get  $dL/dt$  multiplied by a (very simple) function of  $L$  on one side, and integrate both sides with respect to time. Alternatively, you may be able to use your knowledge of the derivatives of various functions to guess the solution, which has a simple dependence on  $t$ . (c) Will the water eventually freeze to the bottom of the flask?

**128.** As the very first rudiment of climatology, estimate the temperature of Earth. Assume it is a perfect sphere and its temperature is uniform. Ignore the greenhouse effect. Thermal radiation from the Sun has an intensity (the "solar constant"  $S$ ) of about  $1370 \text{ W/m}^2$  at the radius of Earth's orbit. (a) Assuming the Sun's rays are parallel, what area must  $S$  be multiplied by to get the total radiation intercepted by Earth? It will be easiest to answer in terms of Earth's radius,  $R$ . (b) Assume that Earth reflects about 30% of the solar energy it intercepts. In other words, Earth has an albedo with a value of  $A = 0.3$ . In terms of  $S$ ,  $A$ , and  $R$ , what is the rate at which Earth absorbs energy from the Sun? (c) Find the temperature at which Earth radiates energy at the same rate. Assume that at the infrared wavelengths where it radiates, the emissivity  $e$  is 1. Does your result show that the greenhouse effect is important? (d) How does your answer depend on the the area of Earth?

**129.** Let's stop ignoring the greenhouse effect and incorporate it into the previous problem in a very rough way. Assume the atmosphere is a single layer, a spherical shell around Earth, with an emissivity  $e = 0.77$  (chosen simply to give the right answer) at infrared wavelengths emitted by Earth and by the atmosphere. However, the atmosphere is transparent to the Sun's radiation (that is, assume the radiation is at visible wavelengths with no infrared), so the Sun's radiation reaches the surface. The greenhouse effect comes from the difference between the atmosphere's transmission of visible light and its rather strong absorption of infrared. Note that the atmosphere's radius is not significantly different from Earth's, but since the atmosphere is a layer above Earth, it emits radiation both upward and downward, so it has twice Earth's area. There are three radiative energy transfers in this problem: solar radiation absorbed by Earth's surface; infrared radiation from the surface, which is absorbed by the atmosphere according to its emissivity; and infrared radiation from the atmosphere, half of which is absorbed by Earth and half of which goes out into space. Apply the method of the previous problem to get an equation for Earth's surface and one for the atmosphere, and solve them for the two unknown temperatures, surface and atmosphere.

- a. In terms of Earth's radius, the constant  $\sigma$ , and the unknown temperature  $T_s$  of the surface, what is the power of the infrared radiation from the surface?
- b. What is the power of Earth's radiation absorbed by the atmosphere?
- c. In terms of the unknown temperature  $T_e$  of the atmosphere, what is the power radiated from the atmosphere?
- d. Write an equation that says the power of the radiation the atmosphere absorbs from Earth equals the power of the radiation it emits.
- e. Half of the power radiated by the atmosphere hits Earth. Write an equation that says that the power Earth absorbs from the atmosphere and the Sun equals the power that it emits.
- f. Solve your two equations for the unknown temperature of Earth.

For steps that make this model less crude, see for example [the lectures \(https://openstaxcollege.org//21paulgormlec\)](https://openstaxcollege.org//21paulgormlec) by Paul O'Gorman.